UCDAVIS

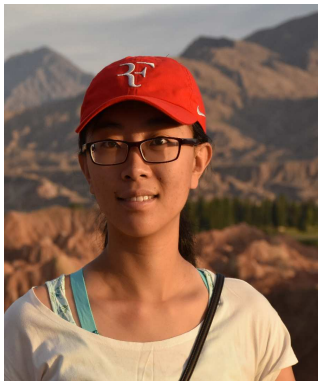
Is Perfect Quantum State Transfer Possible?

- The spreading of wave packets
- Lattice Models
- Perfect Quantum State Transfer
- Monte Carlo and the Inverse Eigenvalue problem
- Cavity-Emitter Systems and Some Results
- Conclusions

Funding:

U.S. DEPARTMENT OF ENERGY | Office of Science

Cast of Characters



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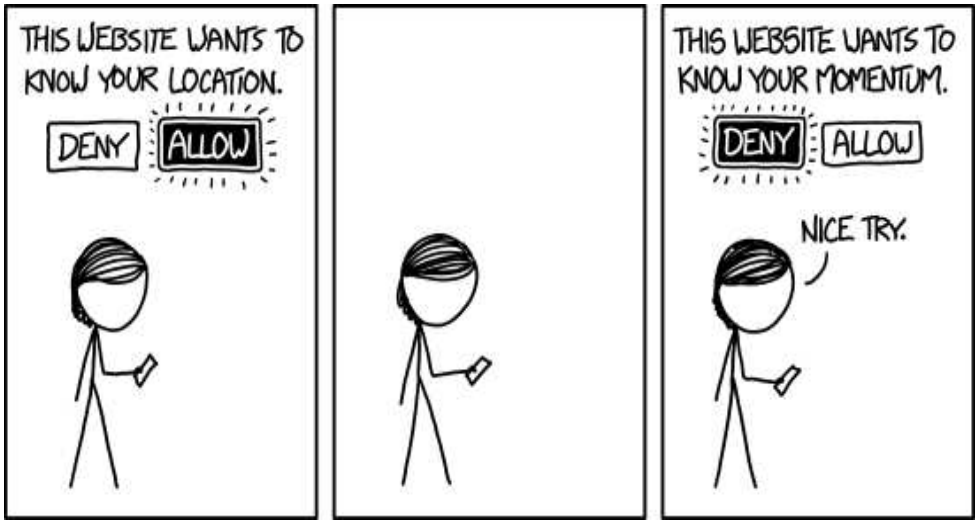
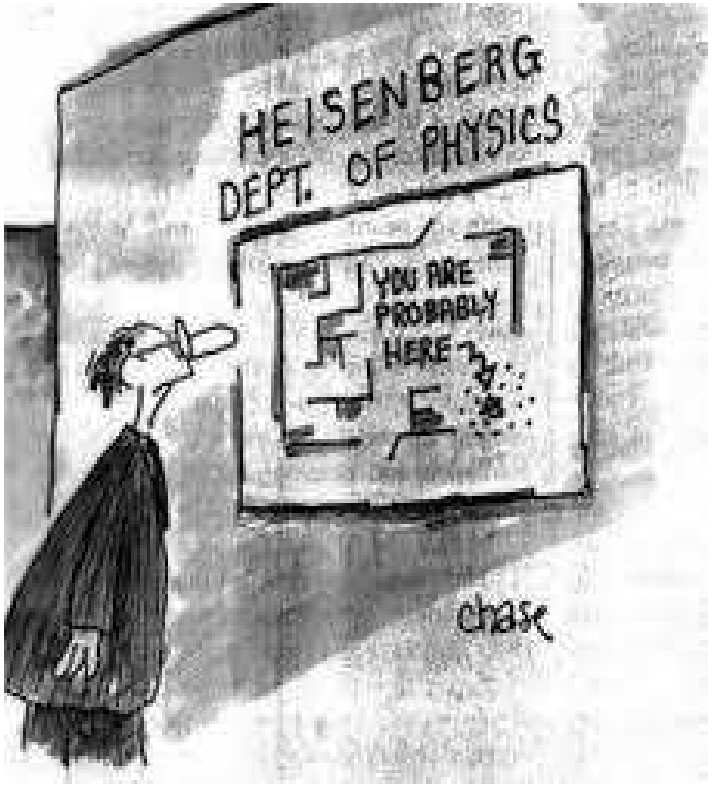
1. The Spreading of Wave Packets

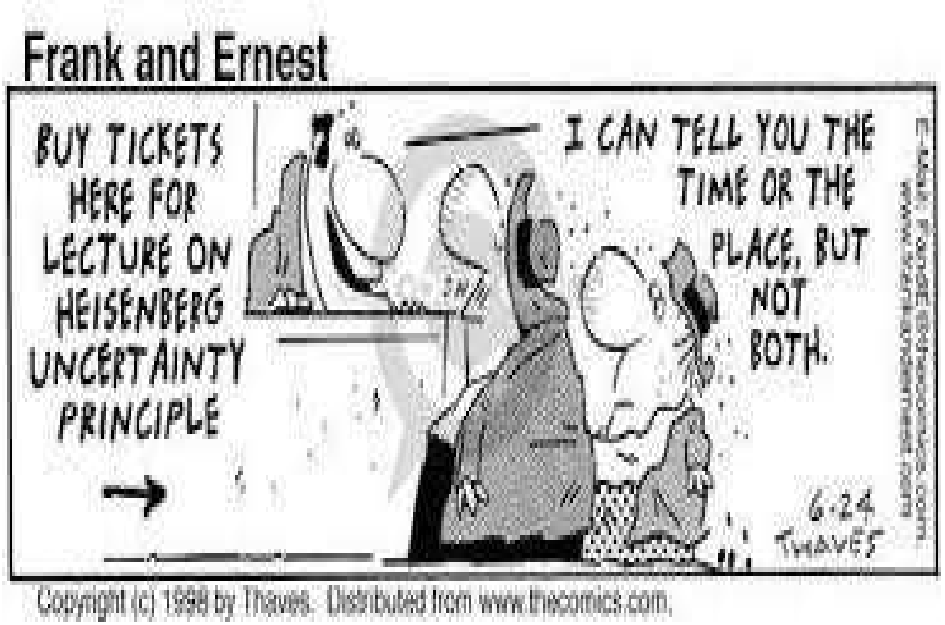
Heisenberg uncertainty principle has become part of popular culture.





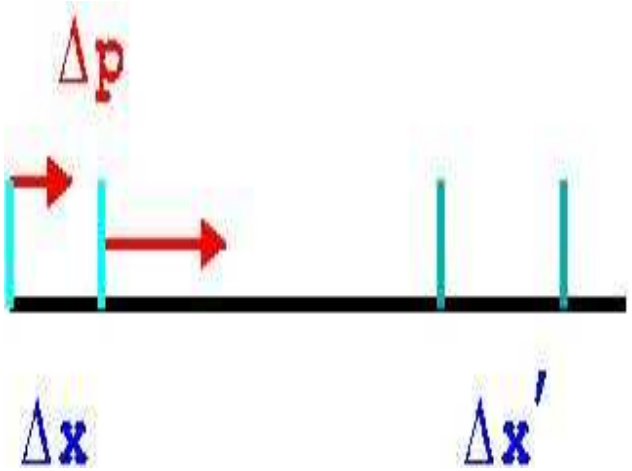
Heisenberg could have prevented your attendence of this colloquium $\cdots$

Cartoons can get it wrong.



$\Delta x\, \Delta t > $?!?!

Uncertainty $\Delta x \Delta p > \frac{\hbar}{2}$, of static $\Psi(x)$: wavefunction also spreads in time.



This picture useful qualitatively,

but, like cartoon, also wrong.

Would imply linear in t growth of Δx.

Doing it right: free-particle Schroedinger Equation

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

'Imaginary time' diffusion equation

$$\frac{\partial \rho(x,t)}{\partial t} = D \frac{\partial^2 \rho(x,t)}{\partial x^2}$$

(This analogy underlies a powerful computational approach to the solution of the Schroedinger equation: "diffusion monte carlo")

Videos of numerical solution of Schroedinger equation.

`https://scalettar.physics.ucdavis.edu/p104a/V10atx80K04new.gif`

External potential can control spreading: e.g. Hydrogen atom

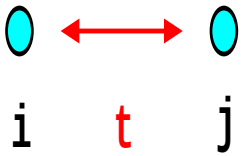
$$\frac{\hbar}{i} \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\, \psi(x,t)$$

2. Lattice Models

Diffusion, localization and quantum state transfer often formulated on a lattice.

'Second quantized' notation: $c_i^\dagger (c_i)$ create(destroy) particles.

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right)$$



Little fundamental difference from continuum space.

Combine 'Bloch states' $\phi_k(j) = \frac{1}{\sqrt{N}} e^{ikj}$ (eigenstates of $\mathcal{H}$ of eigenvalue ϵ_k)

$$\psi(j, t=0) = \sum_k c_k \, \phi_k(j) \qquad\qquad \psi(j,t) = \sum_k c_k \, e^{-i\epsilon_k t/\hbar} \, \phi_k(j)$$

to form wave packets which can propagate and spread.

Anderson localization: Nuclear potential confines electron in an atom $\Rightarrow$

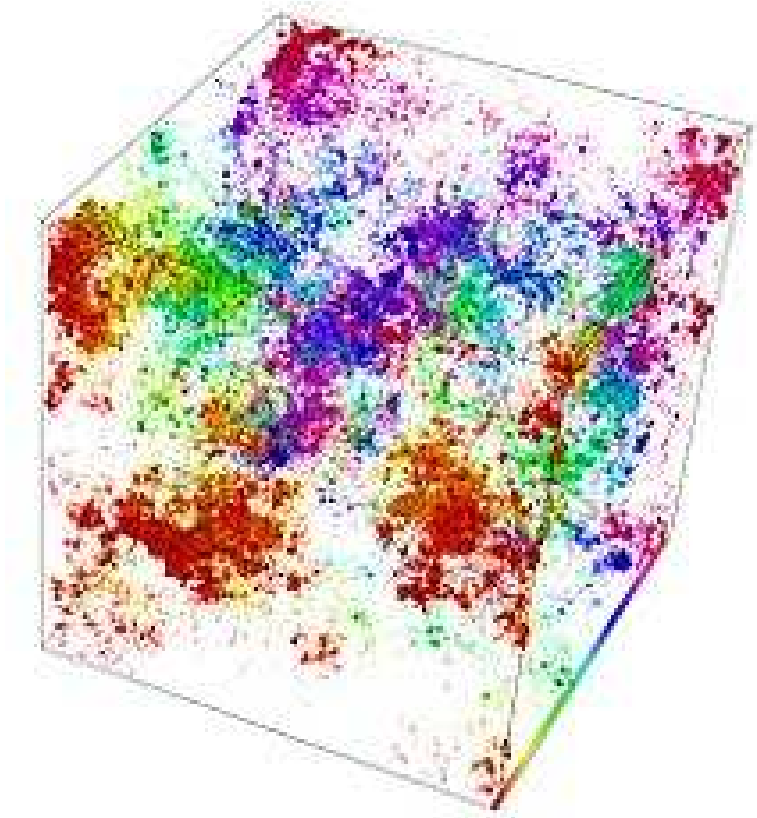
Site energies μ_i localize quantum particles on lattice sites i of lowest μ_i.

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i \mu_i \, c_i^\dagger c_i$$

Quantify 'size' of ψ via inverse participation ratio: $\mathcal{I} \equiv \sum_j |\psi_j|^4$

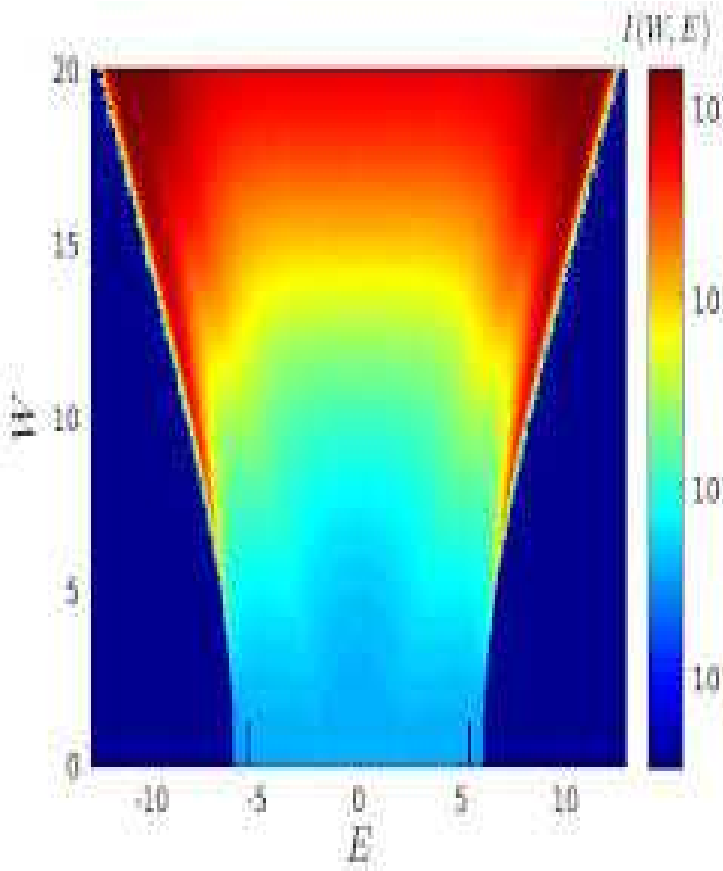
$$\psi(j) = \delta_{j,j_0} \quad \Rightarrow \quad \mathcal{I} = 1 \qquad\qquad \psi(j) = \frac{1}{\sqrt{N}} \quad \Rightarrow \quad \mathcal{I} = \frac{1}{N}$$

$\mathcal{I}$ is inverse of number of sites 'participating' in wave function ψ.



Some eigenstates of the Anderson model in 3D.

$\mathcal{I}$ as function of disorder $|\mu_i| < W$ and eigenenergy E for 3D Anderson model.



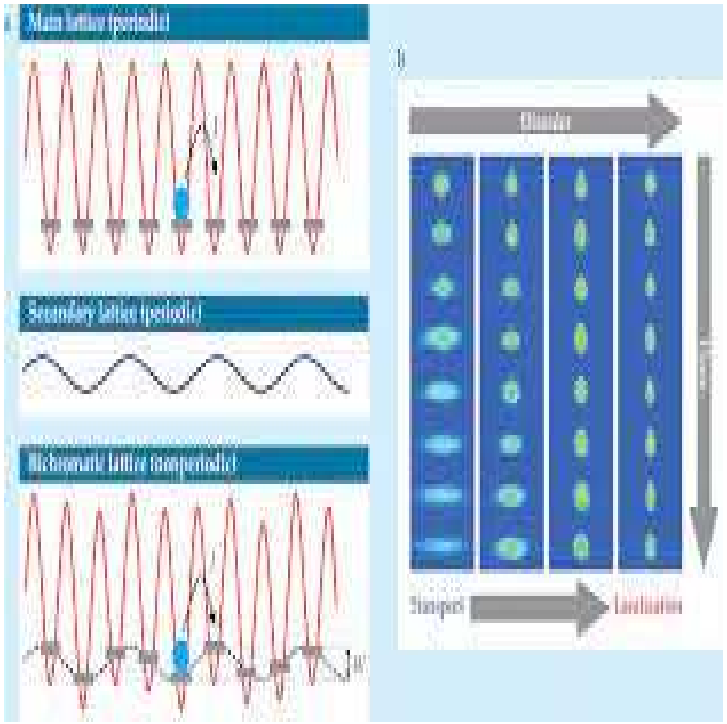
$\to$ 10 sites.

$\to 10^2$ sites.

$\to 10^3$ sites.

$\to 10^4$ sites.

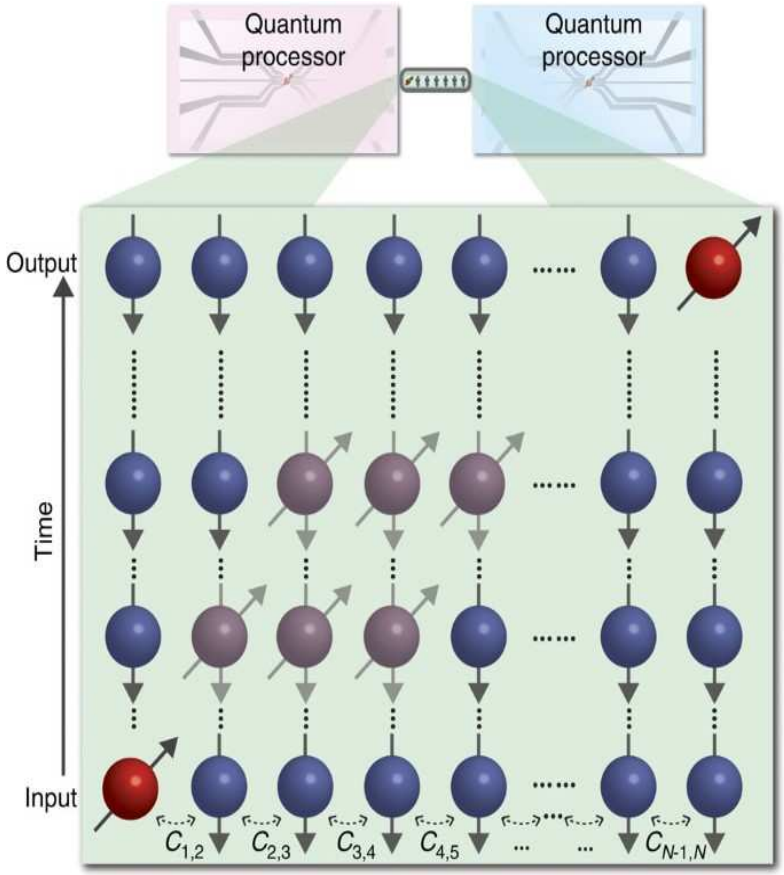
Anderson localization of cold atoms in optical lattice:



(Dark blue regions: no eigenstates.)

Mobility Edge ($\sim$ yellow) separates localized/delocalized states.

3. Perfect Quantum State Transfer



In designing a quantum computer,
or other quantum information
applications, spreading is very bad news.
Would like instead to be able
to transport a quantum state
precisely from one location to another.

Is this goal at variance with our intuition concerning the Schroedinger equation?

After all, imaginary time *diffusion* equation.

Can we engineer a lattice Hamiltonian exhibiting perfect quantum state transfer?

Revisit:

$$\mathcal{H} = -\sum_{\langle ij \rangle} t_{ij} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i \mu_i c_i^\dagger c_i$$

Cleverly tune $\{ t_{ij}, \mu_i \}$ to engineer eigenstates ϕ_α and eigenenergies E_α of $\mathcal{H}$.

Goal: At some passage time t_p

$$\psi(j, t = 0) = \sum_\alpha c_\alpha \, \phi_\alpha(j) = \delta_{j,1} \qquad \Rightarrow \qquad \psi(j, t_p) = \sum_\alpha c_\alpha \, e^{-i\epsilon_\alpha t_p/\hbar} \, \phi_\alpha(j) = \delta_{j,N}$$

Is this possible?!

Intuition: Eigen-energies E_α must allow ψ to be 'in phase' at later time t.

$E_\alpha - E_\beta$ related as *rational fractions*. Simplest scenario: $E_\alpha - E_\beta = c$.

Do we know any quantum mechanical system with equi-spaced eigenenergies?

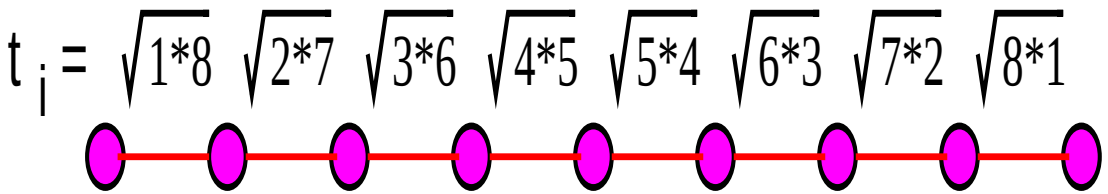
We sure do! Quantum harmonic oscillator.

Crud! That's a infinite collection $\Rightarrow$ infinite length chain.

Ah-ha. Angular momentum J has $J_z = m = \hbar(-j, -j+1, \cdots j)$

$$J_+ \; |j, m\rangle = \sqrt{j(j+1) - m(m+1)} \; |j, m+1\rangle$$

$j = 4$ has nine $m = -4, -3, -2, -1, 0, 1, 2, 3, 4$.

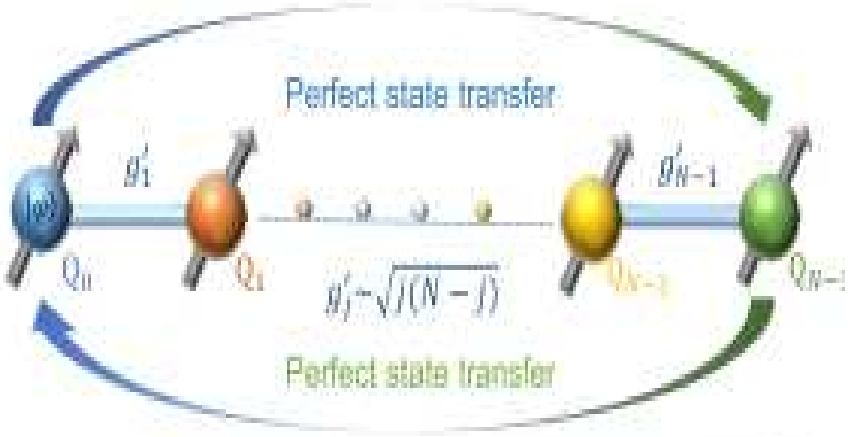


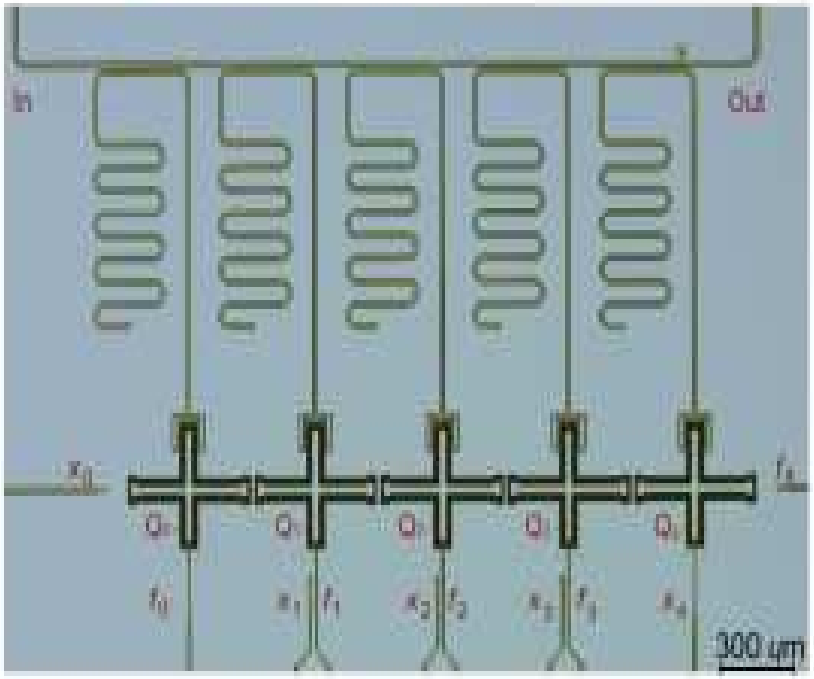
Spin Chain: 'engineered' hoppings (for $N = 9$) which will give perfect QST!

Passage time: $t_p = \frac{\pi}{2}$.

Symmetry $t_i = t_{N-i}$ will be important. Notice too: No μ_i (as yet).

These ‘quantum spin chain’ perfect state transfer systems are being built!





“Perfect quantum state transfer in a superconducting qubit chain with parametrically tunable couplings”,
X. Li, *etal*,
Phys. Rev. Applied 10, 054009 (2018).

Five Qubits.

Well-studied problem.
“Perfect transfer of arbitrary states in quantum spin networks”,
M. Christandl etal,
Phys. Rev. A71 032312 (2005).

We will be interested in more complex geometries.

4. Monte Carlo and the Inverse Eigenvalue Problem

George Marsaglia (1924-2011) was an American mathematician and computer scientist.

Professor Emeritus of Pure and Applied Mathematics and Computer Science at Washington State University.

Established the lattice structure of linear congruential generators: G. Marsaglia, "Random numbers fall mainly in the planes", Proc. Natl. Acad. Sci. 61, 25 (1968).

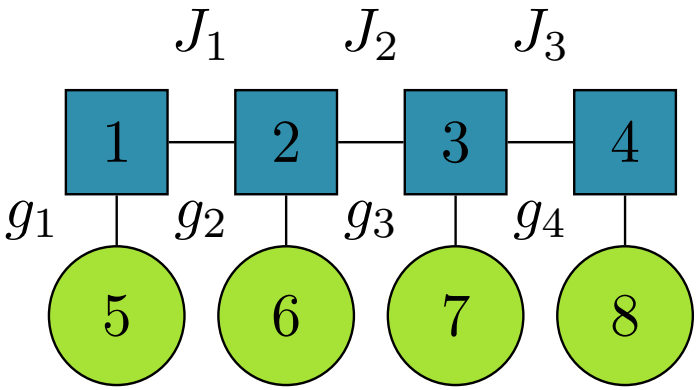
Marsaglia, G., Tsang, W.W., "The Monty Python method for generating random variables," ACM Transactions on Mathematical Software, 24, 341 (1998).

Marsaglia called this the Monty Python method because opening graphics on the British television show "Monty Python's Flying Circus" resembled the essential element: The zany Monty Python crew pictured a stylized head with a hinged top that folded open, with all kinds of silliness pouring out.

1D chain with near-neighbor hopping: $\mathcal{H}$ is a tridiagonal matrix (left, below),

$$\mathcal{H} = \begin{pmatrix} 0 & J_1 & 0 & \dots & 0 \\ J_1 & 0 & J_2 & \dots & 0 \\ 0 & J_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & J_{N-1} \\ 0 & 0 & 0 & J_{N-1} & 0 \end{pmatrix} \qquad \mathcal{H} = \begin{pmatrix} \Omega_1 & J_1 & 0 & 0 & g_1 & 0 & 0 & 0 \\ J_1 & \Omega_2 & J_2 & 0 & 0 & g_2 & 0 & 0 \\ 0 & J_2 & \Omega_3 & J_3 & 0 & 0 & g_3 & 0 \\ 0 & 0 & J_3 & \Omega_4 & 0 & 0 & 0 & g_4 \\ g_1 & 0 & 0 & 0 & \omega_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & 0 & \omega_2 & 0 & 0 \\ 0 & 0 & g_3 & 0 & 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & g_4 & 0 & 0 & 0 & \omega_4 \end{pmatrix}$$

We will be interested in this geometry ($N = 4$):



$\mathcal{H}$ (for $N = 4$) at right, above.

Notes:

- Allow non-zero diagonal entries.
- Notation change: $t_i \to J_i$.
- Ω_i, ω_i in a moment $\cdots$

Similar problem: Engineer $\{ J_i, g_i \}$ for $\{ \lambda_\alpha \}$ with appropriate relationships.

Proceed via Monte Carlo.

Assume eigenvalues for original $d = 1$ geometry (length $2N$) giving perfect QST also give perfect QST for new 'two-component' $N + N$ geometry.

'Target' eigenvalues $\lambda_n^{(t)}$. Define an action:

$$\mathcal{S} = \sum_n \left(\lambda_n - \lambda_n^{(t)} \right)^2$$

λ_n: actual eigenvalues of the matrix $\mathcal{H}$ of for given $\{ J_i, g_i \}$.

Begin with a random set of $\{ J_i, g_i \}$.

Propose 'moves' which change $\{ J_i, g_i \}$.

Accept with the 'heat bath' probability $e^{-\beta \Delta \mathcal{S}} \left(1 + e^{-\beta \Delta \mathcal{S}} \right)^{-1}$.

$\Delta \mathcal{S} \equiv$ the change in action from Monte Carlo move.

'Annealing:' β starts at a small value (e.g. $\beta_{\text{initial}} \sim 0.1$).

After L Monte Carlo steps increased by factor α.

Repeated for K steps until $\beta_{\text{final}} = \alpha^K \beta_{\text{initial}}$ is large (e.g. $\beta_{\text{final}} = 10^4$.)

Statistical mechanics language $\beta = 1/T$ is the inverse temperature.

$\beta_{\text{initial}} = 0.1$ corresponds to high temperature.

$\beta_{\text{final}} = 10^4$ corresponds to low temperature.

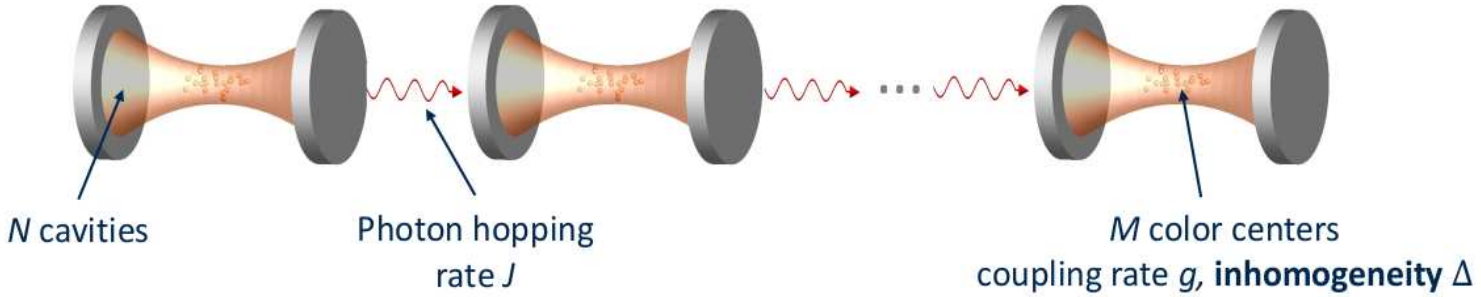
Gradual increase of β (lowering of T) allows Monte Carlo to escape metastable states.

'Ground state' $\mathcal{S} = 0$:

$\{J_i, g_i\}$ give Hamiltonian with target eigenvalues to high accuracy.

5. Cavity-Emitter Systems and Some Results

Where does this geometry come from?



Coupled Cavity Array (CCA): chain of optical cavities.

Light localized on the nanometer scale.

Photons hop between adjacent cavities due to overlap of neighboring resonance modes.

CCA may contain atom-like 'emitters' coupled to cavity electromagnetic field.

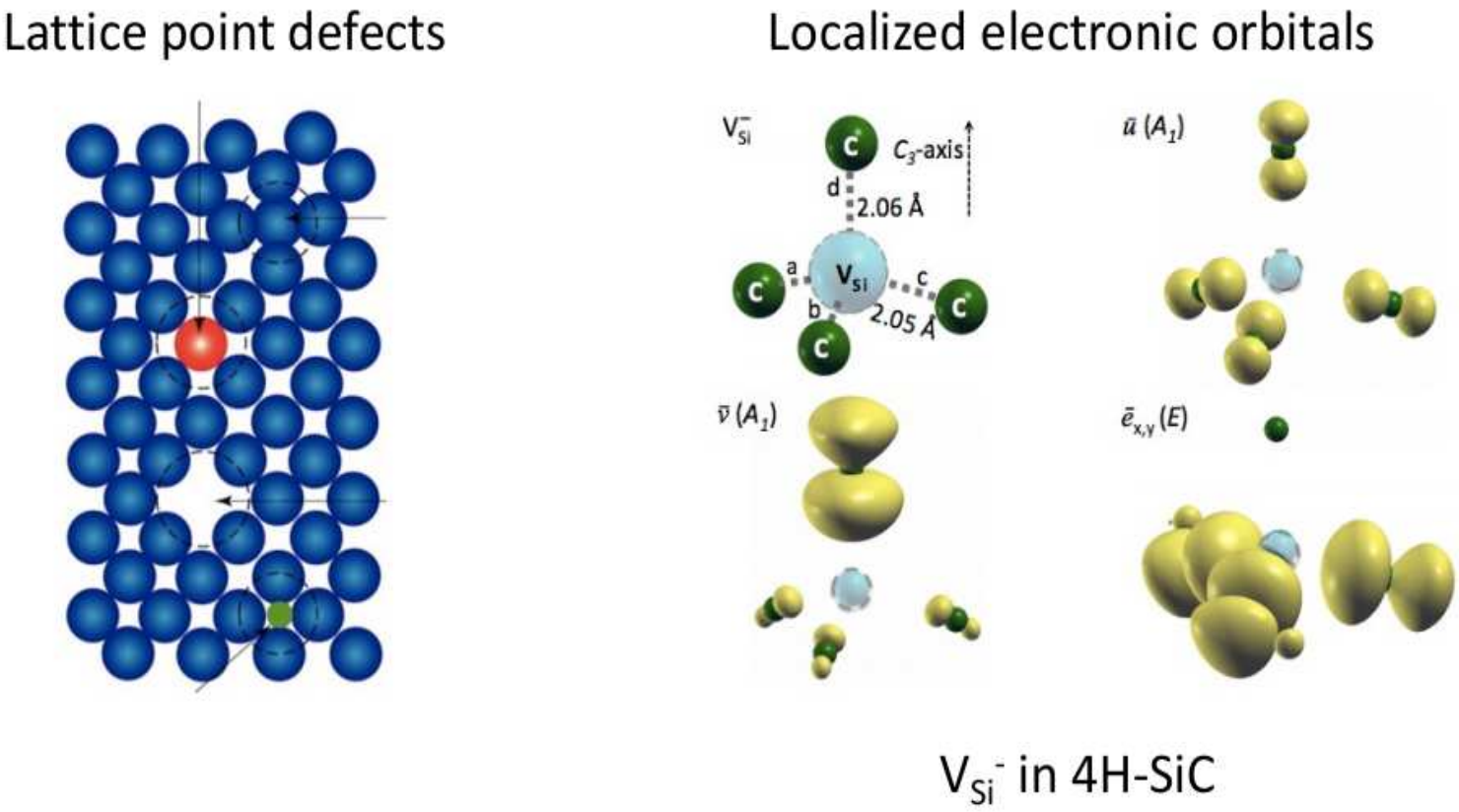
Expect variance in cavity levels Ω_i, atomic levels ω_i, number of atoms/cavity.

Strong interactions between light and matter can be induced.

Superfluid to insulator transitions and other many-body phenomena examined.

"Quantum simulation with interacting photons",
M.J. Hartmann, Journal of Optics 18, 104005 (2016).

Emitters (Quasi-atoms): color centers formed as lattice defects in semi-conductors.



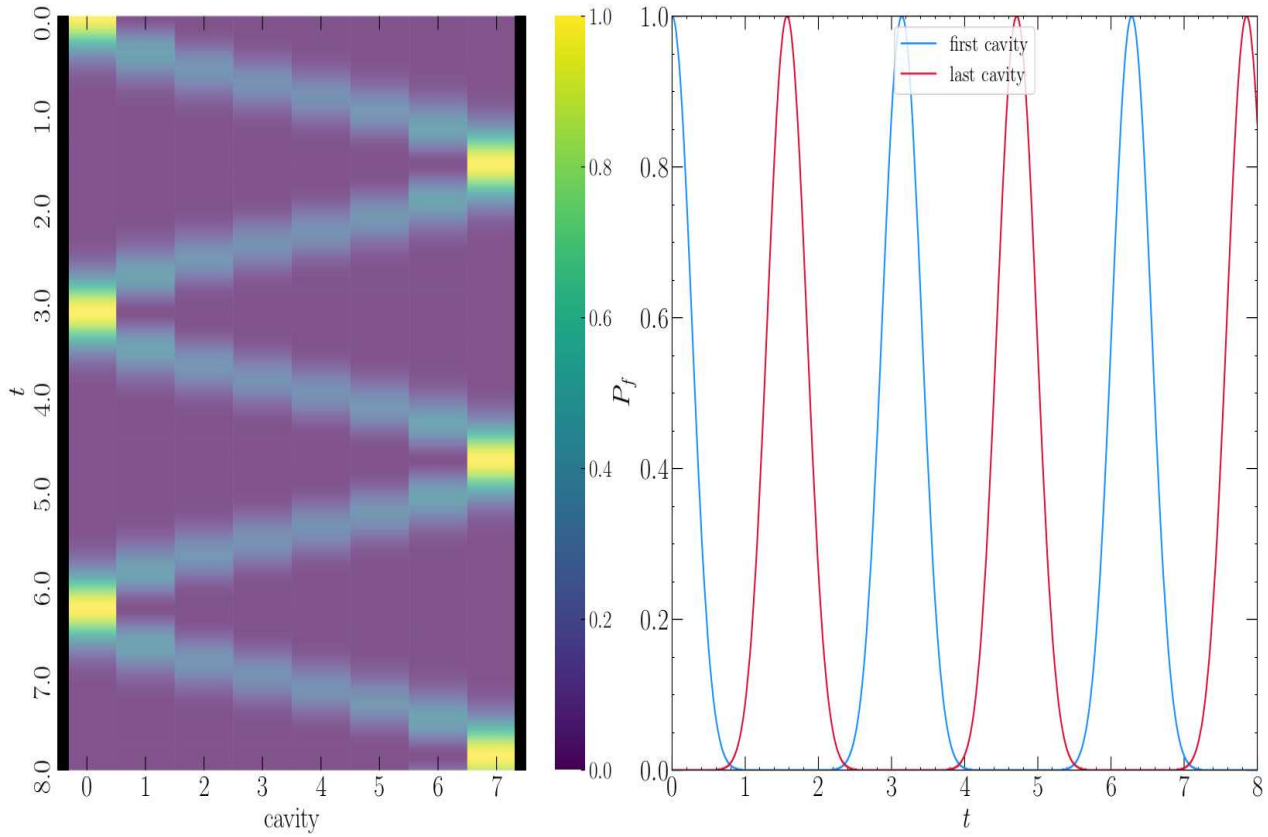
Defect causes electron wavefunctions to localize, creating isolated energy levels.

Most common material substrates: silicon carbide and diamond.

"Silicon Carbide and Color Center Quantum Photonics", M. Radulaski, (2017).

"Quantum photonics in triangular-cross-section nanodevices in silicon carbide", S. Majety *etal*, Journal of Physics: Photonics 3, 034008 (2021).

Perfect Quantum State Transfer for the original 1D ('spin chain') geometry:

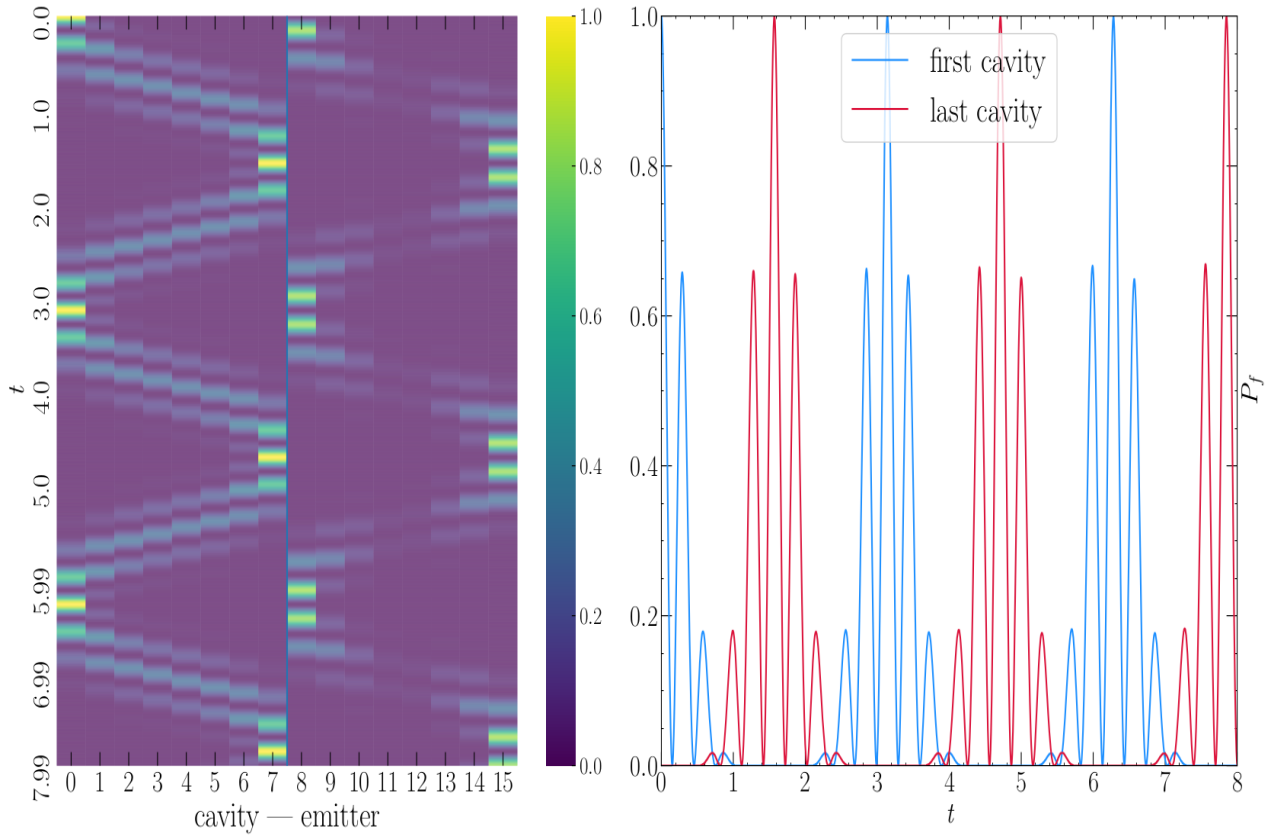


Monte Carlo converges to analytically known (angular momentum) eigenvalues.

(Verifying known result.)

Return to initial site $i = 1$: Reflection symmetry of $\mathcal{H}$.

Perfect Quantum State Transfer for the CCA geometry:

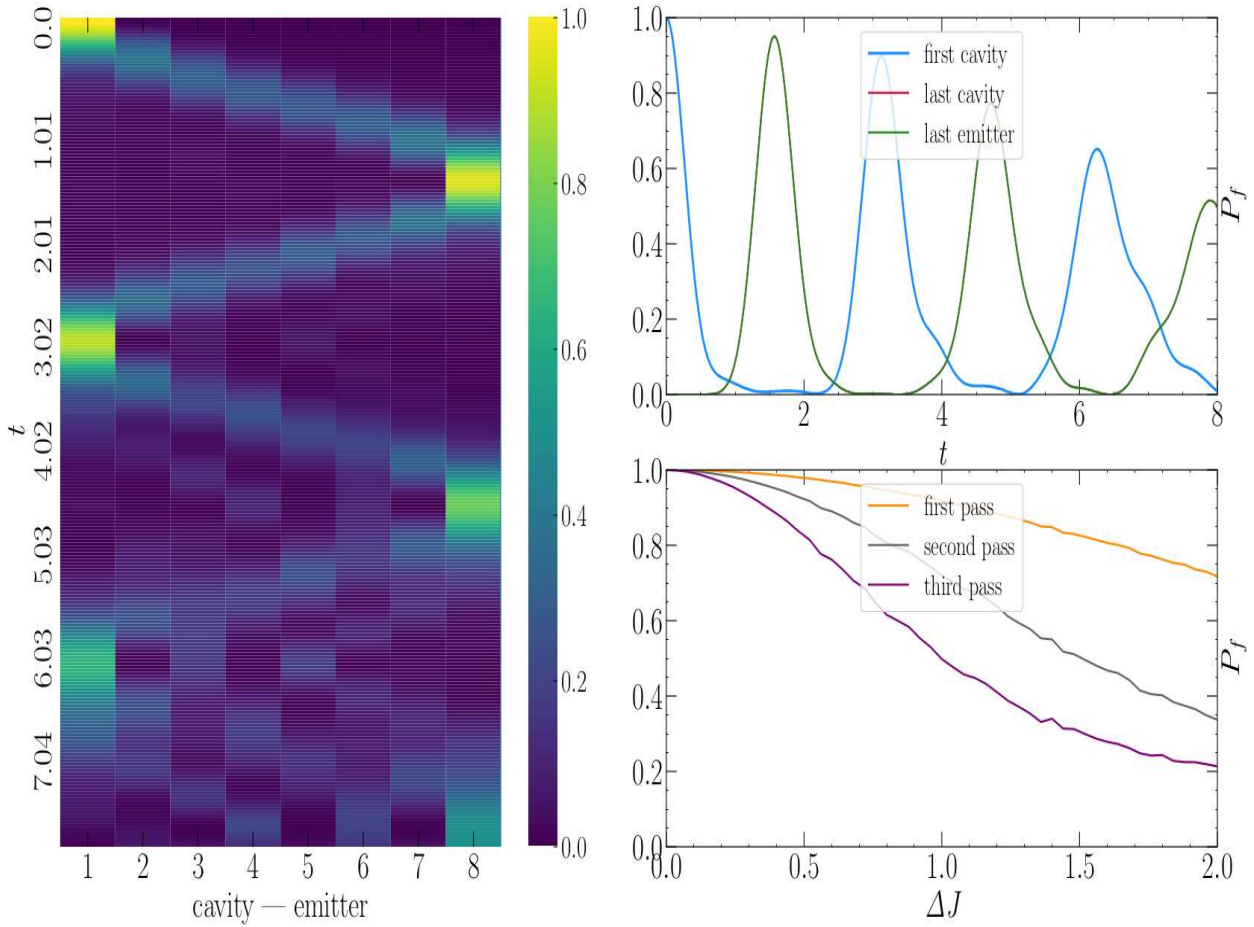


Monte Carlo works! Transfer with perfect fidelity from site $i = 1$ to site $i = N$.

Small/negligible deviation from fidelity $f = 1$ due to finite MC simulation time.

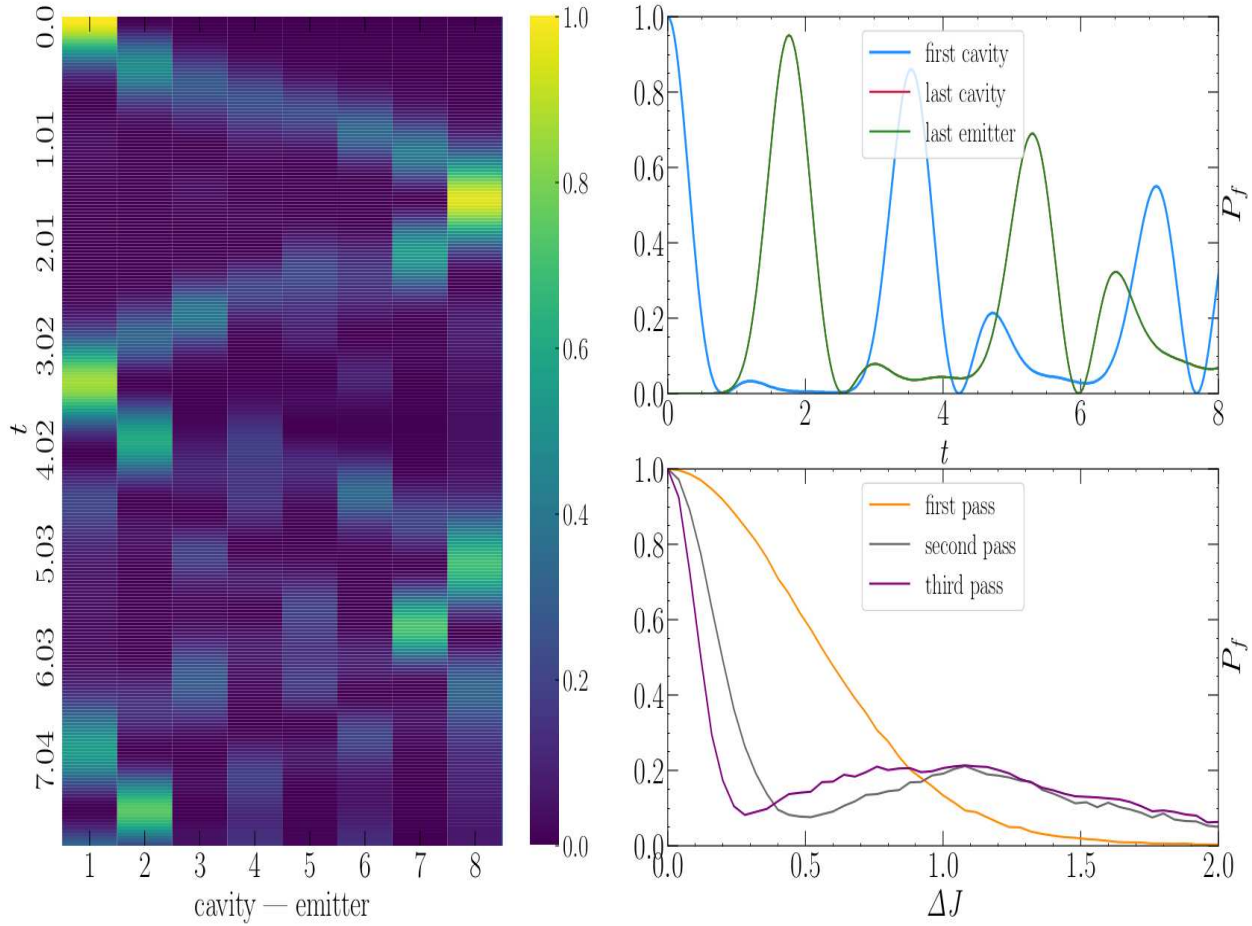
Can achieve arbitrary accuracy by lengthening run.

Real materials are imperfect: Effect of Energy (Ω_i) Disorder.



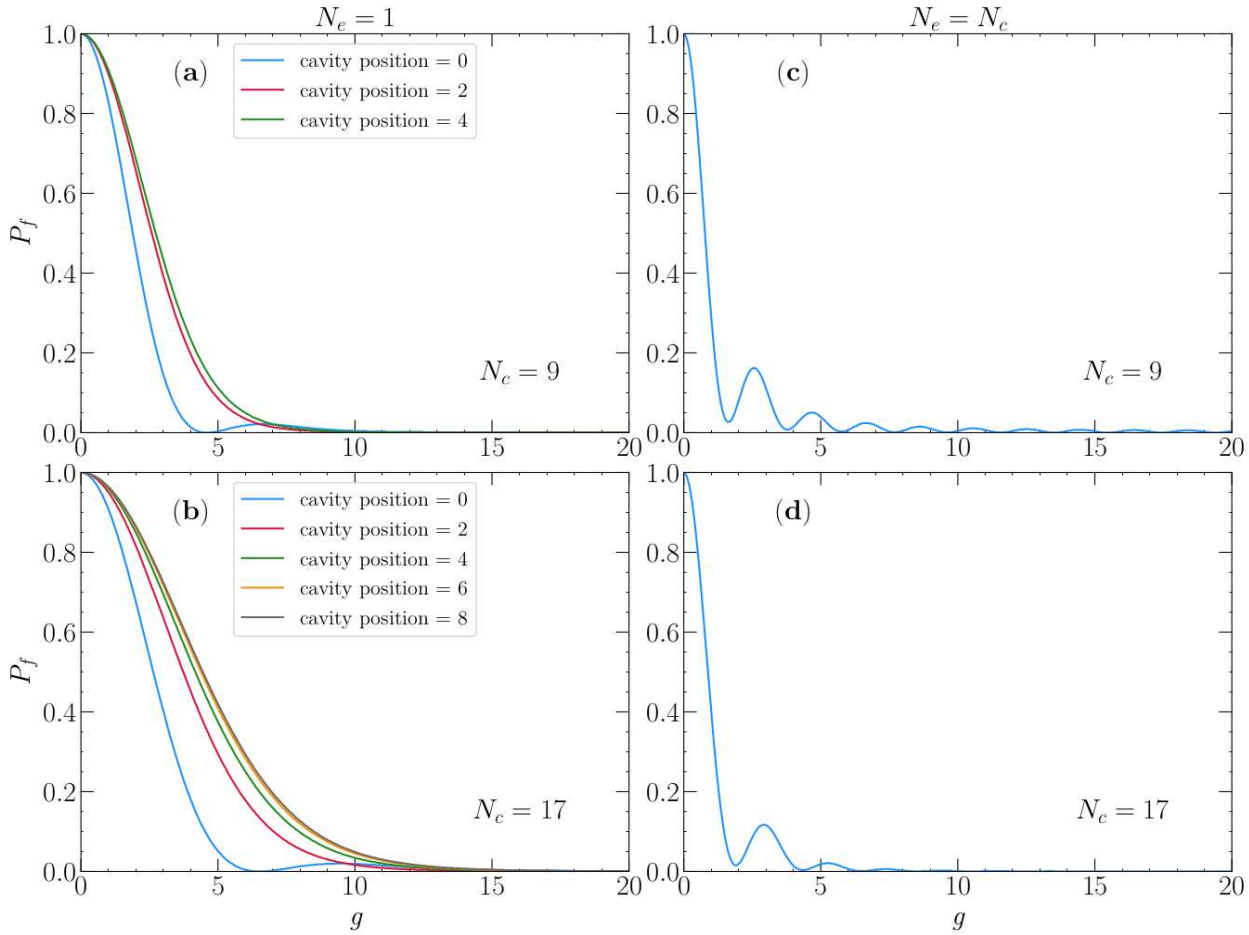
Gradual loss of fidelity quantified.

Real materials are imperfect: Effect of hopping (t_i) Disorder.



Gradual loss of fidelity quantified.

Real materials are imperfect: Effect of hopping random emitter positions.



Random $\{J_i\}$ effect on QST previously explored.

Novel type of 'geometric disorder'.

6. Conclusions

- Usual diffusion of wave function can be circumvented by 'engineering'.
- Monte Carlo method useful for inverse eigenvalue problem.
- Extended known 'spin chain' results to CCA.
- Quantification of 'geometric disorder' on fidelity loss.

To learn more: Michael Forbes

Dynamics of quantum many-body systems (include interactions!)

Typical experimental CCA parameters:

$g_i \sim 5\,\mathrm{GHz}$	$0 < \Delta g_i < g_{\max}$
$J_i \sim 1\,\mathrm{GHz}$	can be 'repaired'
$\Omega_i \sim 200\,\mathrm{THz}$	can be 'repaired'

Systems that have been constructed:

- ~ 20 empty cavities (no emitters)
- One cavity with two emitters
- Two cavities with one emitter each

Origins of disorder:

- Δg_i position of emitter within cavity.
- $\Delta \omega_i$ variation of strain in material
- $\Delta J_i, \Delta \Omega_i$ can be 'repaired' (laborious) photo-oxidation of part of cavity.