UCDAVIS

Is Perfect Quantum State Transfer Possible?

- The spreading of wave packets
- Lattice Models
- Theory of Perfect Quantum State Transfer
  (vs. The Real World)
- Recovering high fidelity with Monte Carlo
- Superconducting Qubit Array Results (w. Marina: Cavity-Emitter)
- Conclusions

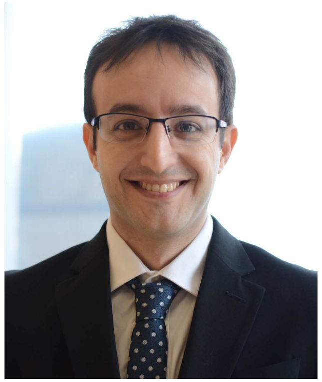
Funding:

U.S. DEPARTMENT OF ENERGY | Office of Science

Cast of Characters

Yuxi Zhang

Marina Radulaski



Rubem Mondaini

Amelia Broman

Jesse Patton

Qiujiang Guo

Eli Baum

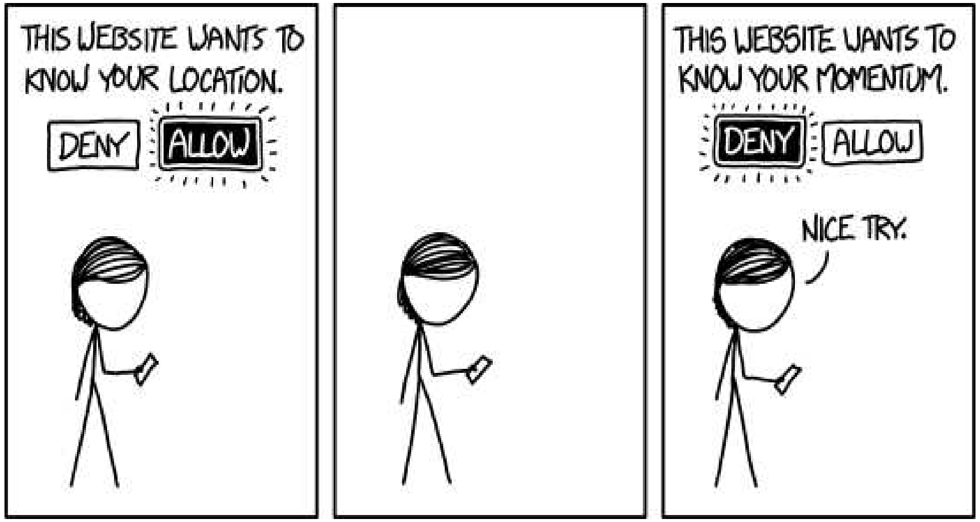
Victoria Norman

Trevor Clarke

Alex Yue

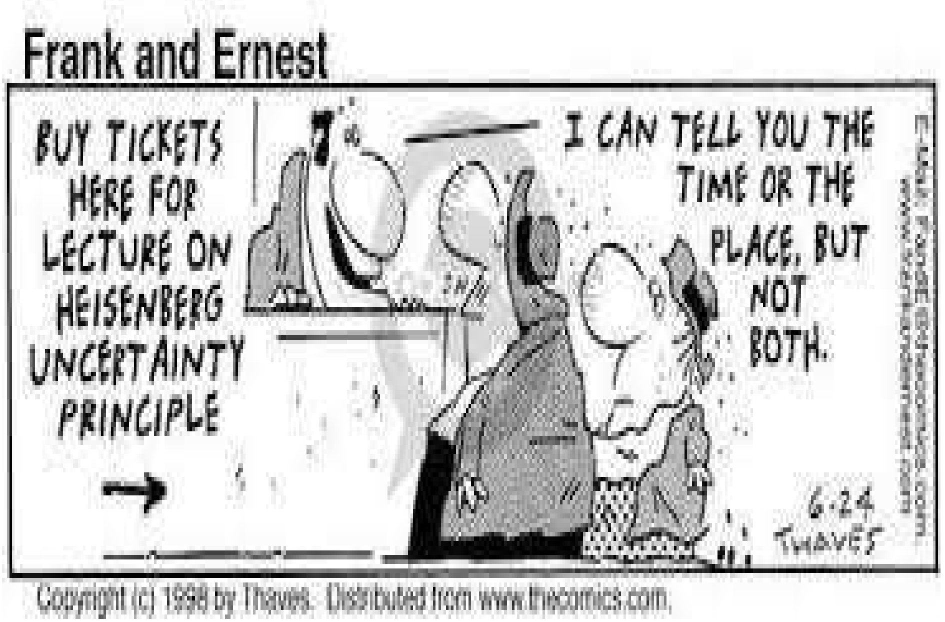
1. The Spreading of Wave Packets

Heisenberg uncertainty principle has become part of popular culture.



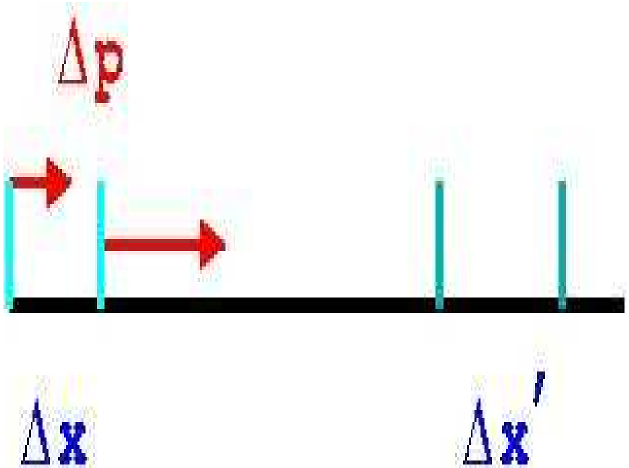
Heisenberg could have prevented your attendence of this talk · · ·

Cartoons can get it wrong.



$\Delta x \, \Delta t > ?!?!$

Uncertainty $\Delta x \Delta p > \frac{\hbar}{2}$, of static $\Psi(x)$: wavefunction also spreads in time.



This picture useful qualitatively,
but, like cartoon, also wrong.
Would imply linear in t growth of Δx.

Doing it right: free-particle Schroedinger Equation

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

'Imaginary time' diffusion equation

$$\frac{\partial \rho(x,t)}{\partial t} = D \frac{\partial^2 \rho(x,t)}{\partial x^2}$$

QM probability density spreads as $\sqrt{t}$.

(This analogy underlies a powerful computational approach to the solution of the Schroedinger equation: "diffusion Monte Carlo")

External potential can control spreading: e.g. Hydrogen atom

$$\frac{\hbar}{i} \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\, \psi(x,t)$$

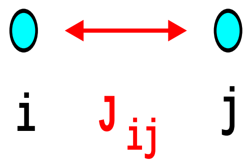
But in the absence of $V(x)$ we expect spreading.

2. Lattice Models

Diffusion, localization and quantum state transfer often formulated on a lattice.

$\sigma_i^{\pm}$ raise/lower a qubit state on site i

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij} \left(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^- \right)$$



Little fundamental difference from continuum space.

Diagonalize Hamiltonian: eigenvectors $\phi_\alpha(j)$ eigenvalues λ_α

$$\psi(j, t = 0) = \sum_\alpha c_\alpha \, \phi_\alpha(j) \qquad \psi(j, t) = \sum_\alpha c_\alpha \, e^{-i\lambda_\alpha t/\hbar} \, \phi_\alpha(j)$$

to form wave packets which can propagate and spread.

Analog of nuclear potential confining e^- in an atom $\Rightarrow$ Anderson localization:

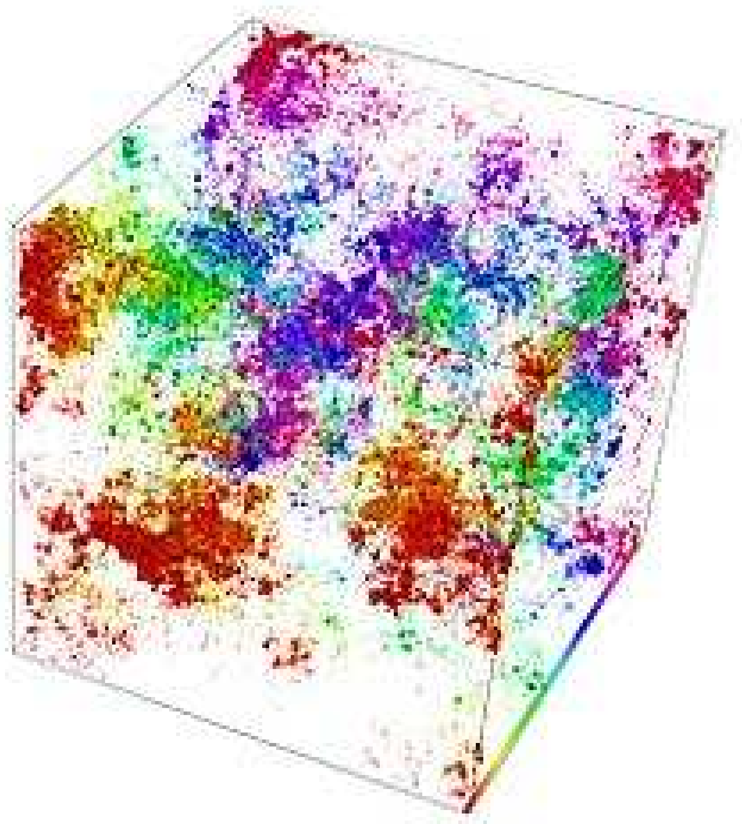
Site energies μ_i localize quantum particles on lattice sites i of lowest μ_i.

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^- \right) + \sum_i \mu_i \, \sigma_i^+ \sigma_i^-$$

Quantify 'size' of ψ via inverse participation ratio: $\mathcal{P}^{-1} \equiv \sum_j \mid \psi_j \mid^4$

$$\psi(j) = \delta_{j,j_0} \quad \Rightarrow \quad \mathcal{P}^{-1} = 1 \qquad\qquad \psi(j) = \frac{1}{\sqrt{N}} \quad \Rightarrow \quad \mathcal{P}^{-1} = \frac{1}{N}$$

$\mathcal{P}^{-1}$ is inverse of number of sites 'participating' in wave function ψ.

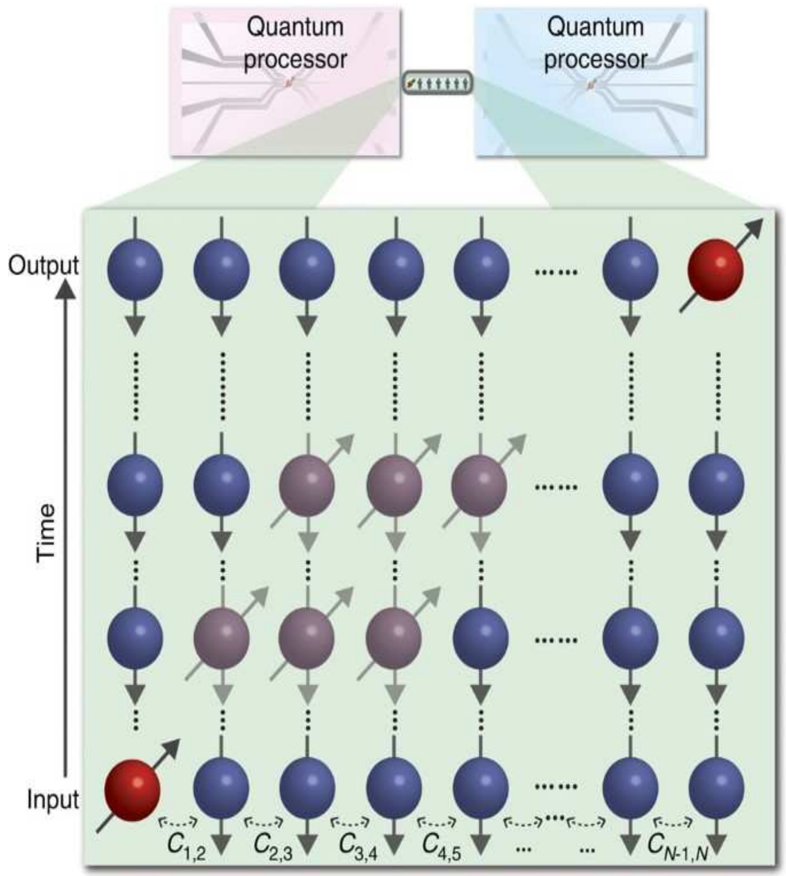


Some eigenstates of the Anderson model in 3D.

Marina's group: generalization of $\mathcal{P}^{-1}$ to cavity-emitter as measure of *polaronicity*.

Do not expect this in translationallty invariant system.

3. Perfect Quantum State Transfer



In designing a quantum computer, or other quantum information applications, spreading is very bad news. Would like instead to be able to transport a quantum state precisely from one location to another.

This goal is at variance with our intuition concerning the Schroedinger equation!

After all, imaginary time *diffusion* equation.

Can we engineer a lattice Hamiltonian exhibiting perfect quantum state transfer?

Revisit:

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i \mu_i c_i^\dagger c_i$$

Tune $\{ J_{ij}, \mu_i \}$ to engineer eigenstates ϕ_α and eigenenergies λ_α of $\mathcal{H}$.

Goal: At some passage time t_p

$$\psi(j, t = 0) = \sum_\alpha c_\alpha \, \phi_\alpha(j) = \delta_{j,1} \qquad \Rightarrow \qquad \psi(j, t_p) = \sum_\alpha c_\alpha \, e^{-i\lambda_\alpha t_p/\hbar} \, \phi_\alpha(j) = \delta_{j,N}$$

Is this possible?!

Intuition: Eigen-energies λ_α must allow ψ to be 'in phase' at later time t.

$\lambda_\alpha - \lambda_\beta$ related as *rational fractions*. Simplest scenario: $\lambda_\alpha - \lambda_\beta = c$.

Do we know any quantum mechanical system with equi-spaced eigenenergies?

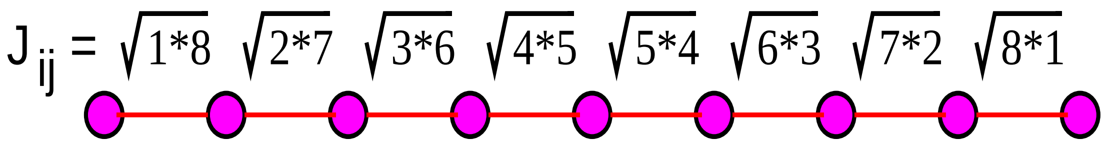
We sure do! Quantum harmonic oscillator.

Crud! That's a infinite collection $\Rightarrow$ infinite length chain.

Ah-ha. Angular momentum J has $J_z = m = \hbar(-j, -j+1, \cdots j)$

$$J_+ \,|j,m\rangle = \sqrt{j(j+1) - m(m+1)}\, |j,m+1\rangle$$

$j = 4$ has nine $m = -4, -3, -2, -1, 0, 1, 2, 3, 4$.



Spin Chain: 'engineered' hoppings (for $N = 9$) which will give perfect QST!

Passage time: $t_p = \frac{\pi}{2}$.

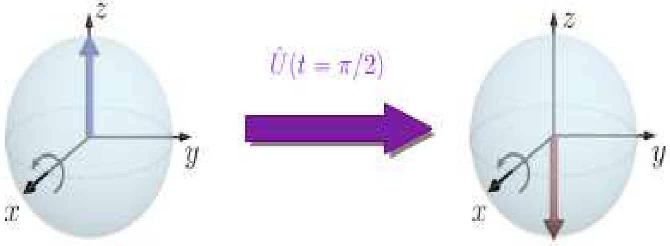
Symmetry $t_i = t_{N-i}$ will be important. Notice too: No μ_i (as yet).

More precisely Christandl says:

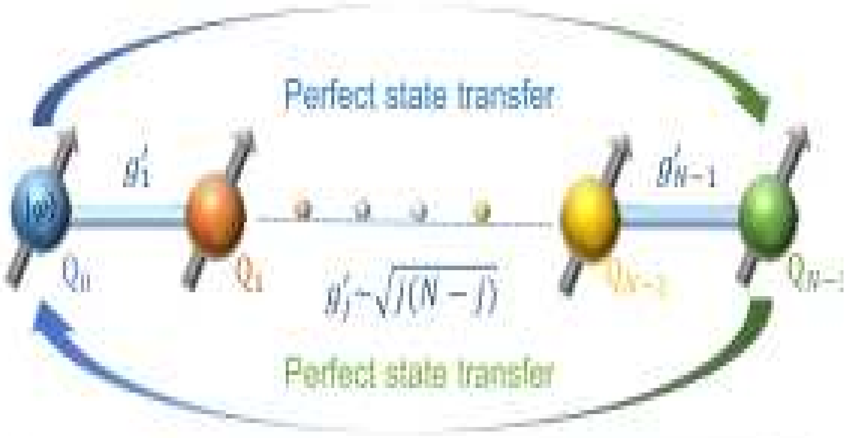
$$\mathcal{H} = \begin{pmatrix} 0 & \sqrt{(N-1)\cdot 1} & 0 & \dots & 0 \\ \sqrt{(N-1)\cdot 1} & 0 & \sqrt{(N-2)\cdot 2} & \dots & 0 \\ 0 & \sqrt{(N-2)\cdot 2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \sqrt{1\cdot(N-1)} \\ 0 & 0 & 0 & \sqrt{1\cdot(N-1)} & 0 \end{pmatrix}$$

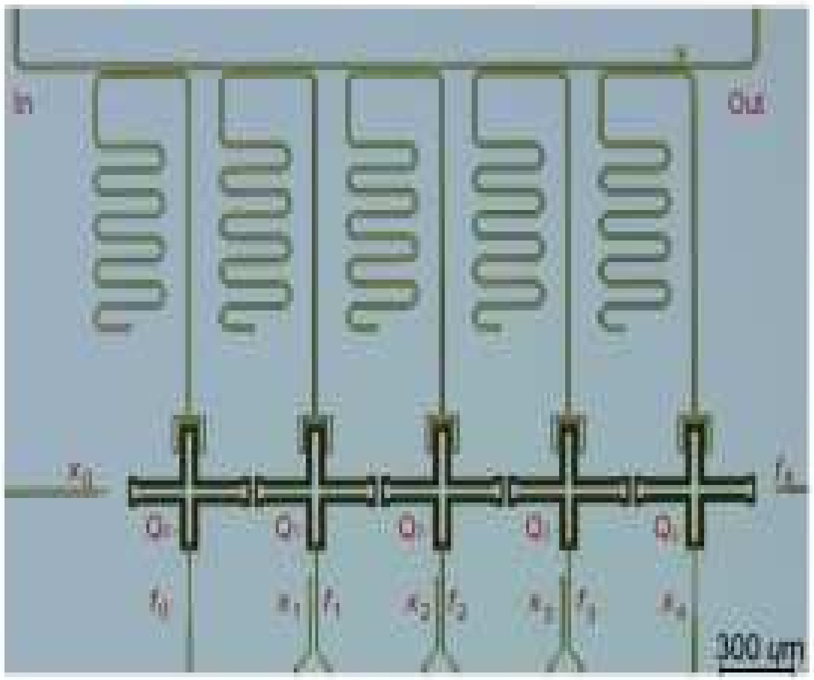
$$\hat{\mathcal{H}} = 2\, J\, \hat{S}_x$$

Time evolution corresponds to rotation of wave function about $\hat{x}$-axis.



These ‘quantum spin chain’ perfect state transfer systems are being built!





Well-studied problem.

“Perfect transfer of arbitrary states in quantum spin networks”,
M. Christandl etal,
Phys. Rev. A71 032312 (2005).

⇒

“Perfect quantum state transfer in a superconducting qubit chain with parametrically tunable couplings”,
X. Li, *etal*,
Phys. Rev. Applied 10, 054009 (2018).

Five Qubits.

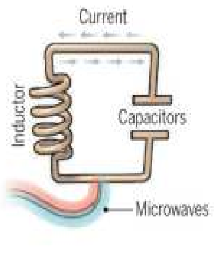
We will be interested in more complex geometries.

3. ⇒ 3'. Real World

[Science, **354** 6316]

Existing **qubit** platforms

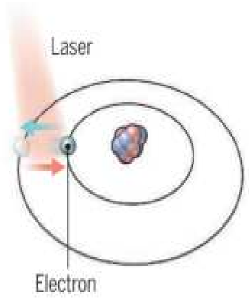
Superconducting loops



- Resistance-free current oscillates back and forth around a circuit loop
- Injected microwave signal excites the current into superposition states
- Emulates a quantum anharmonic oscillator

Google, IBM, …
ZJU, UESTC, ...

Trapped ions



- Ions, have quantum energies that depend on the location of electrons.
- Tuned lasers cool and trap the ions, and put them in superposition states.

IonQ, Honeywell
Maryland, ...

Silicon quantum dots

- "Artificial atoms" made by adding an electron to a small piece of pure silicon.
- Microwaves control the electron's quantum state.

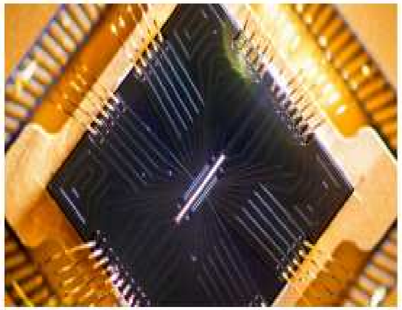
Intel, HRL, QuTech
UNSW, Delft, RIKEN, ...

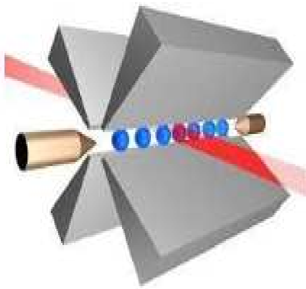
Building many of them – Noisy intermediate quantum devices

Superconducting quantum circuits

[IOP, ZJU]

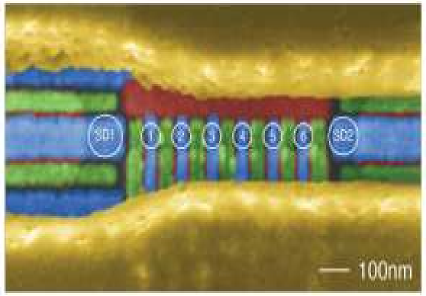
Trapped ions





[Maryland, IonQ]

Silicon quantum dots

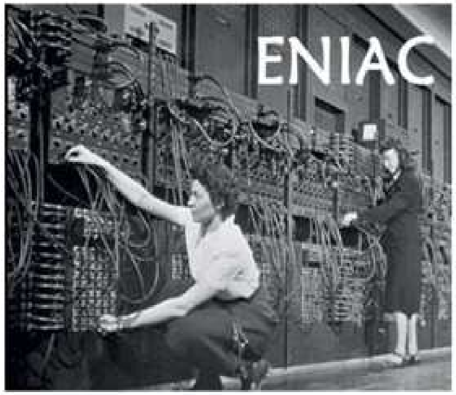


[Qutech 2022]

These are not laptop computers or cell phones ···

The more things change, the more they stay the same...

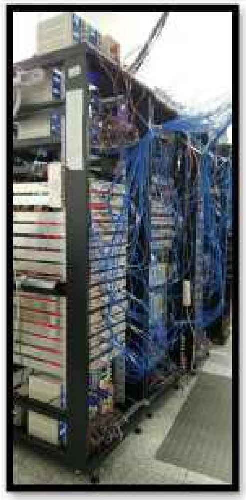
First general-purpose digital computer



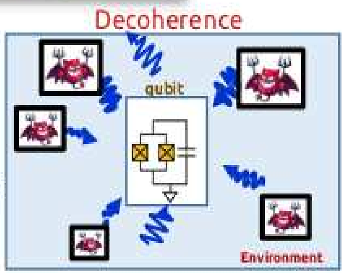
- 30 tons
- 18,000 vacuum tubes
- 1,500 relays
- +100,000 of resistors, capacitors and inductors,

= add or subtract 5,000 times per second!

SC quantum circuit @ZJU



- 36-qubits (121 available)
- Fully programmable
- Emulates dynamics of $\hat{H}$ with dim = 9B states
- Operates at 20mK...

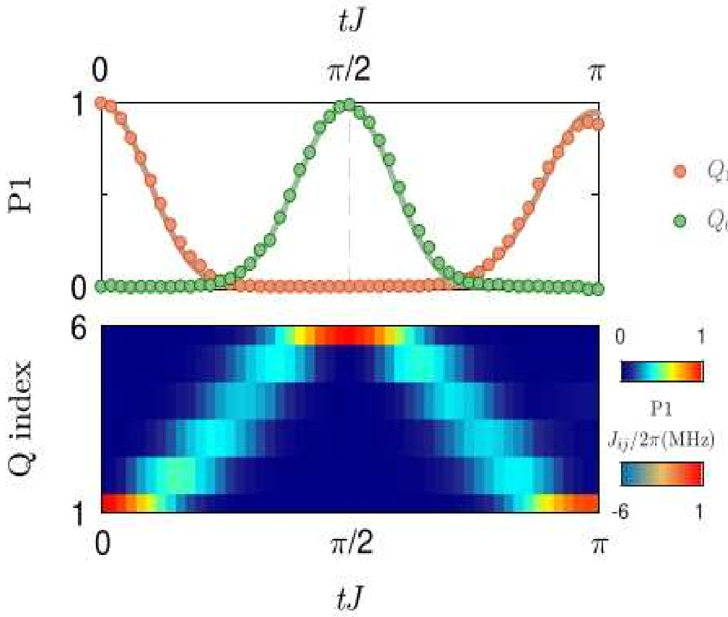


It really works in 1D:

Quantum state transfer: Experimental results

…, **RM***, Guo*, Scalettar*
(in preparation)

1d chain of qubits:



Emulated Hamiltonian (NN couplings are tunable)

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} [\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \sigma_j^+]$$

$$J_{n,n+1} = J\sqrt{n(6-n)}$$

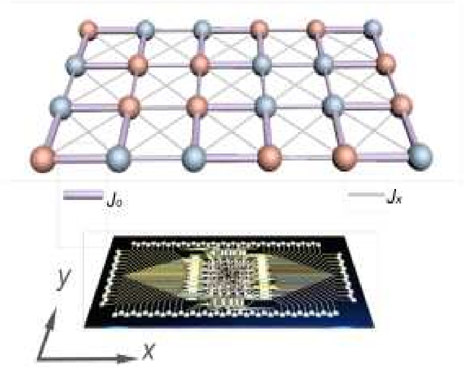
$$J/2\pi = -1 \text{ MHz}$$

- Transfer of one-excitation states with remarkable fidelity

How about different geometries?

3'. Real World Problems

2d quantum state transfer - 3x3



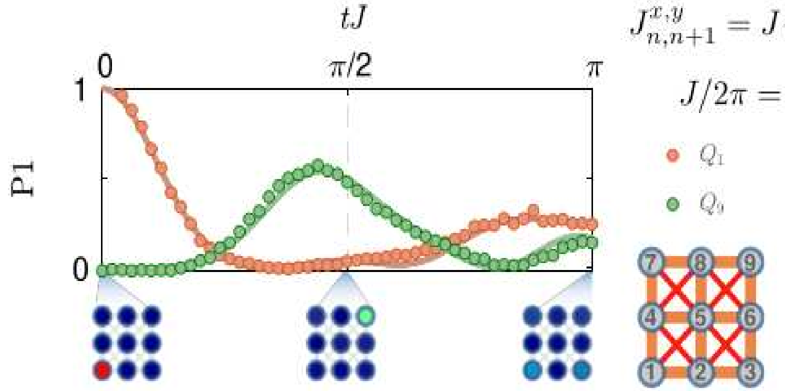
Fabricated @ ZJU

- Parasitic cross couplings $J_\times$ naturally occur in current devices

$$
\begin{aligned}
\hat{H}_{\text{tot}} &= 2J(\hat{S}_{1,x} + \hat{S}_{2,x}) + \\
&\quad J_\times(\hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{1+}\hat{S}_{2-} + \hat{S}_{1+}\hat{S}_{2+} + \hat{S}_{1-}\hat{S}_{2-}) \\
&= 2J(\hat{S}_{1,x} + \hat{S}_{2,x}) + 4J_\times \hat{S}_{1x}\hat{S}_{2x}
\end{aligned}
$$

$$J^{x,y}_{n,n+1} = J\sqrt{n(3-n)}$$

$$J/2\pi = -1 \text{ MHz}$$



➡ spoils "Christandl's prescription"

4. Monte Carlo and the "QST Inverse Problem"

Proceed via Monte Carlo.

Engineer $\{ J_{ij} \}$ to achieve 'Target' time evolution operator

$$\mathcal{U}^* = e^{-i\mathcal{H}^* t}$$

Define an action:

$$\mathcal{S} = \sum_{i,j} \left(\mathcal{U}_{ij} - \mathcal{U}^*_{ij} \right)^2$$

Begin with a random set of $\{ J_{i,j} \}$.

Propose 'moves' which change $\{ J_{i,j} \}$.

Accept with the 'heat bath' probability $e^{-\beta\Delta\mathcal{S}}\left(1 + e^{-\beta\Delta\mathcal{S}}\right)^{-1}$.

$\Delta\mathcal{S} \equiv$ the change in action from Monte Carlo move.

'Annealing:' β starts at a small value (e.g. $\beta_\mathrm{i} \sim 0.1$).

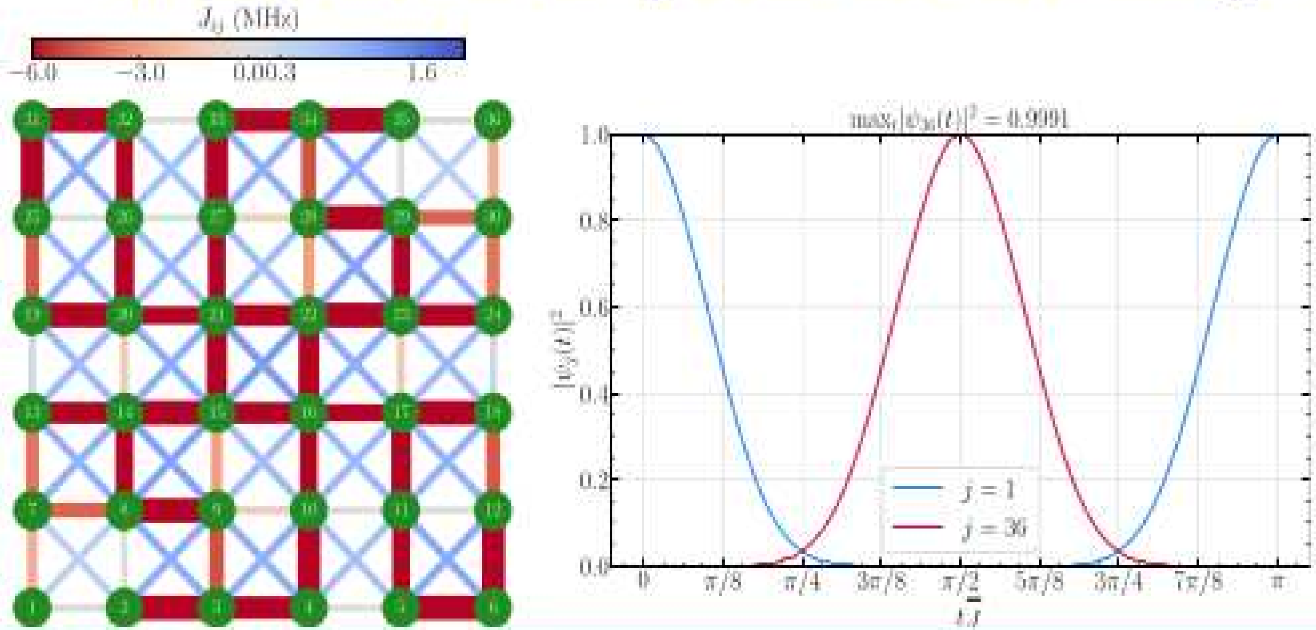
Do Monte Carlo, then increase β. After K steps $\beta_\mathrm{f} = \alpha^K \beta_\mathrm{i}$ (typical $\beta_\mathrm{f} = 10^4$.)

Statistical mechanics language: $\beta = 1/T$ is the inverse temperature.

$\beta_\mathrm{i} = 0.1$: high temperature. $\beta_\mathrm{f} = 10^4$: low temperature. Escape metastable states.

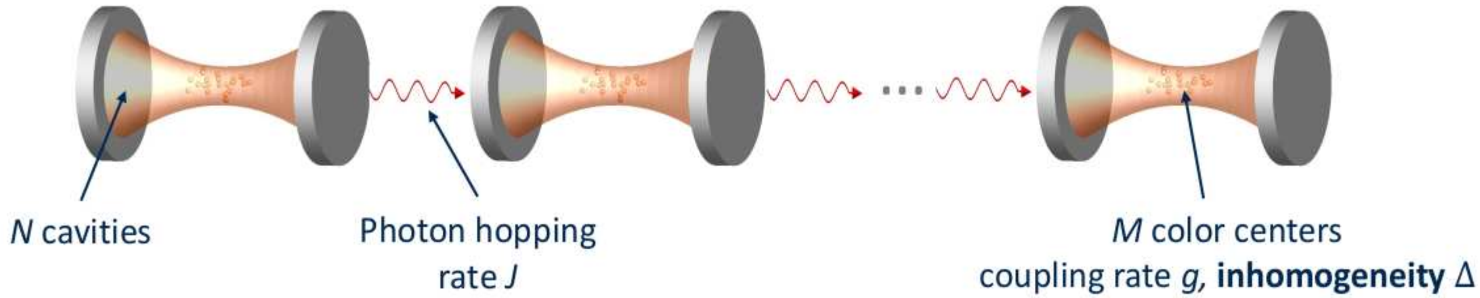
$\{J_i, g_i\}$ give target $\mathcal{U}^*$ high accuracy.

Provides search to configuration that maximizes QST



Similar protocol for coupled cavity-emitter arrays (Radulaski group).

Phys. Rev. B105, 195429 (2022).

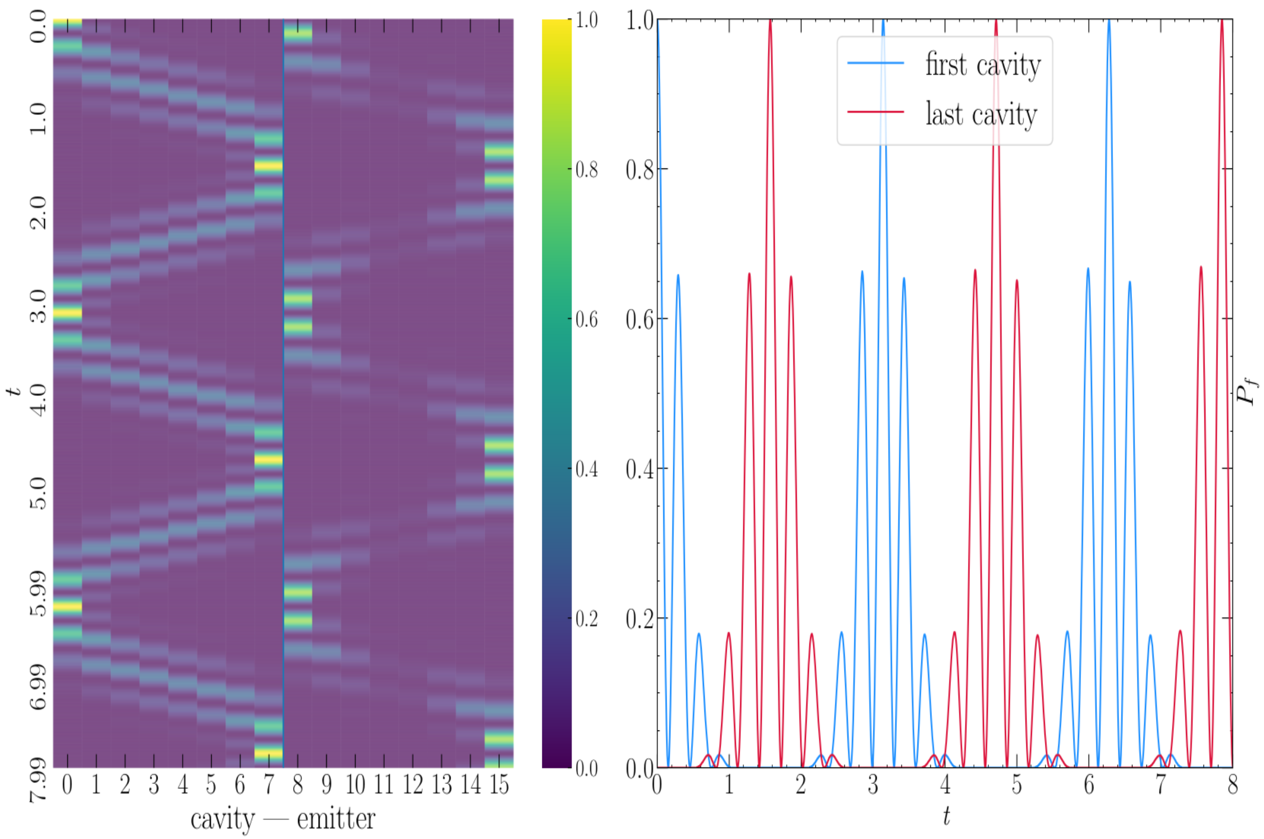


$$\mathcal{H} = \begin{pmatrix} \Omega_1 & J_{1,2} & 0 & 0 & g_1 & 0 & 0 & 0 \\ J_{1,2} & \Omega_2 & J_{2,3} & 0 & 0 & g_2 & 0 & 0 \\ 0 & J_{2,3} & \Omega_3 & J_{3,4} & 0 & 0 & g_3 & 0 \\ 0 & 0 & J_{3,4} & \Omega_4 & 0 & 0 & 0 & g_4 \\ g_1 & 0 & 0 & 0 & \omega_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & 0 & \omega_2 & 0 & 0 \\ 0 & 0 & g_3 & 0 & 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & g_4 & 0 & 0 & 0 & \omega_4 \end{pmatrix}$$

Explored ‘imperfections’ about optimized $\mathcal{H}$.)

- Randomness in J_{ij}, g_i
- Randomness in Ω_i

Perfect Quantum State Transfer for the CCA geometry:



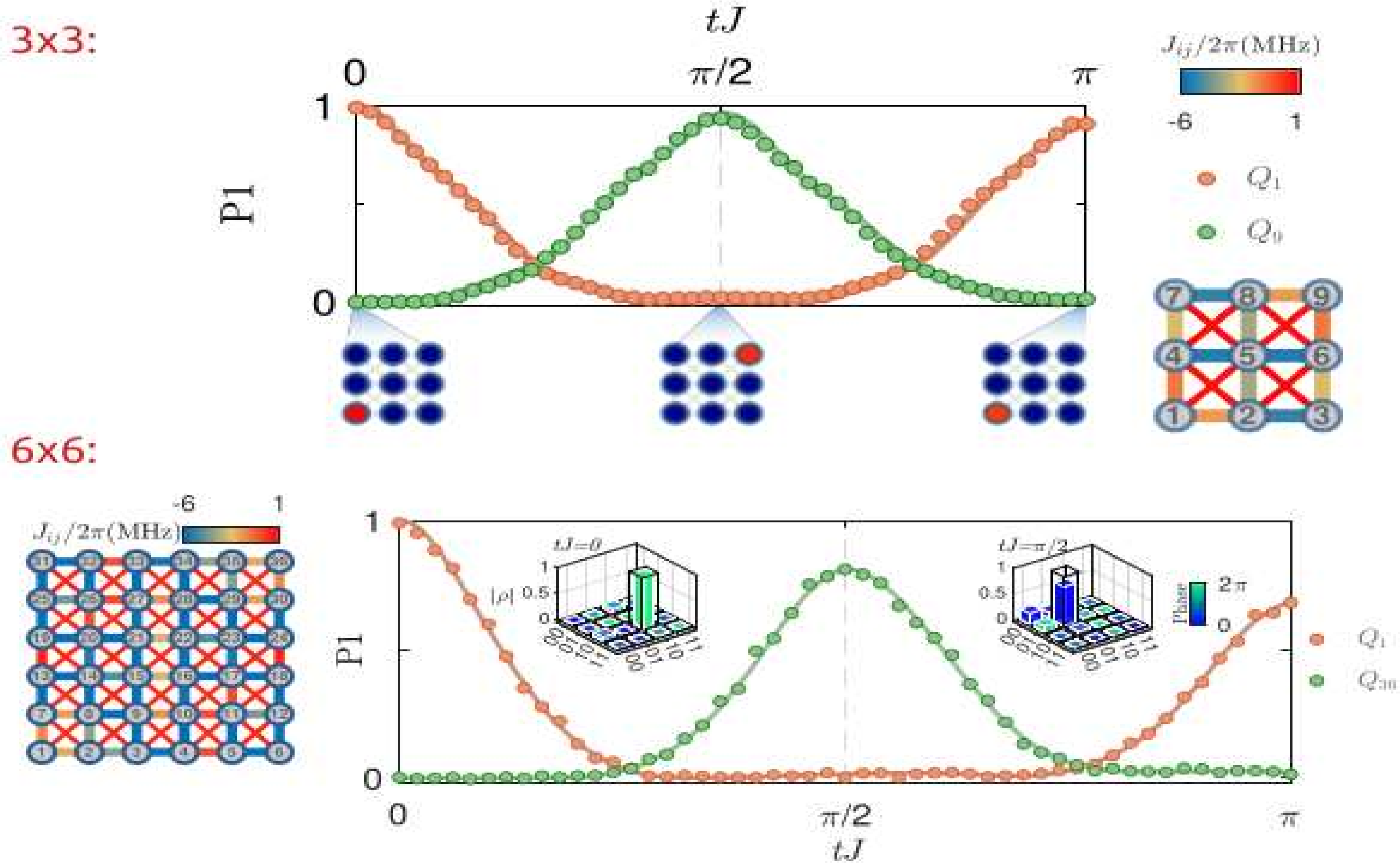
Monte Carlo works! Transfer with perfect fidelity from site $i = 1$ to site $i = N$.

Small/negligible deviation from fidelity $f = 1$ due to finite MC simulation time.

Can achieve arbitrary accuracy by lengthening run.

Can rectify the “real world problems” cross coupling (and defective coupler).

And providing guide to **experiments**!

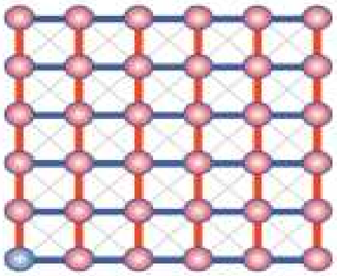


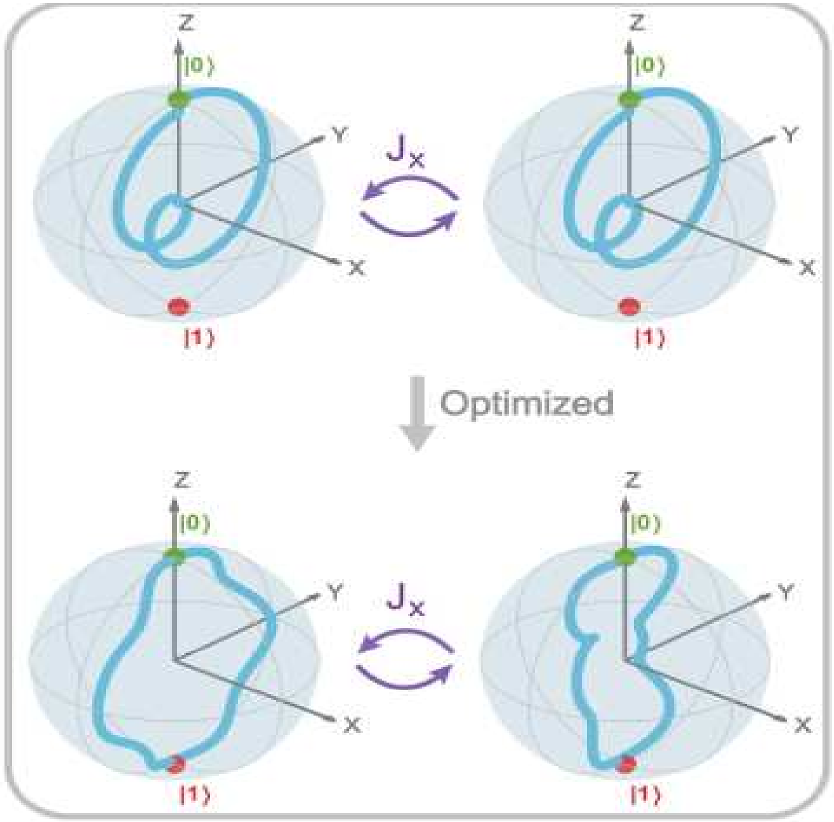
Generalization of Christandl to 2D:

$\hat{S}_x$ and $\hat{S}_y$.

Christandl prescription (top) misses propagation to target qubit.

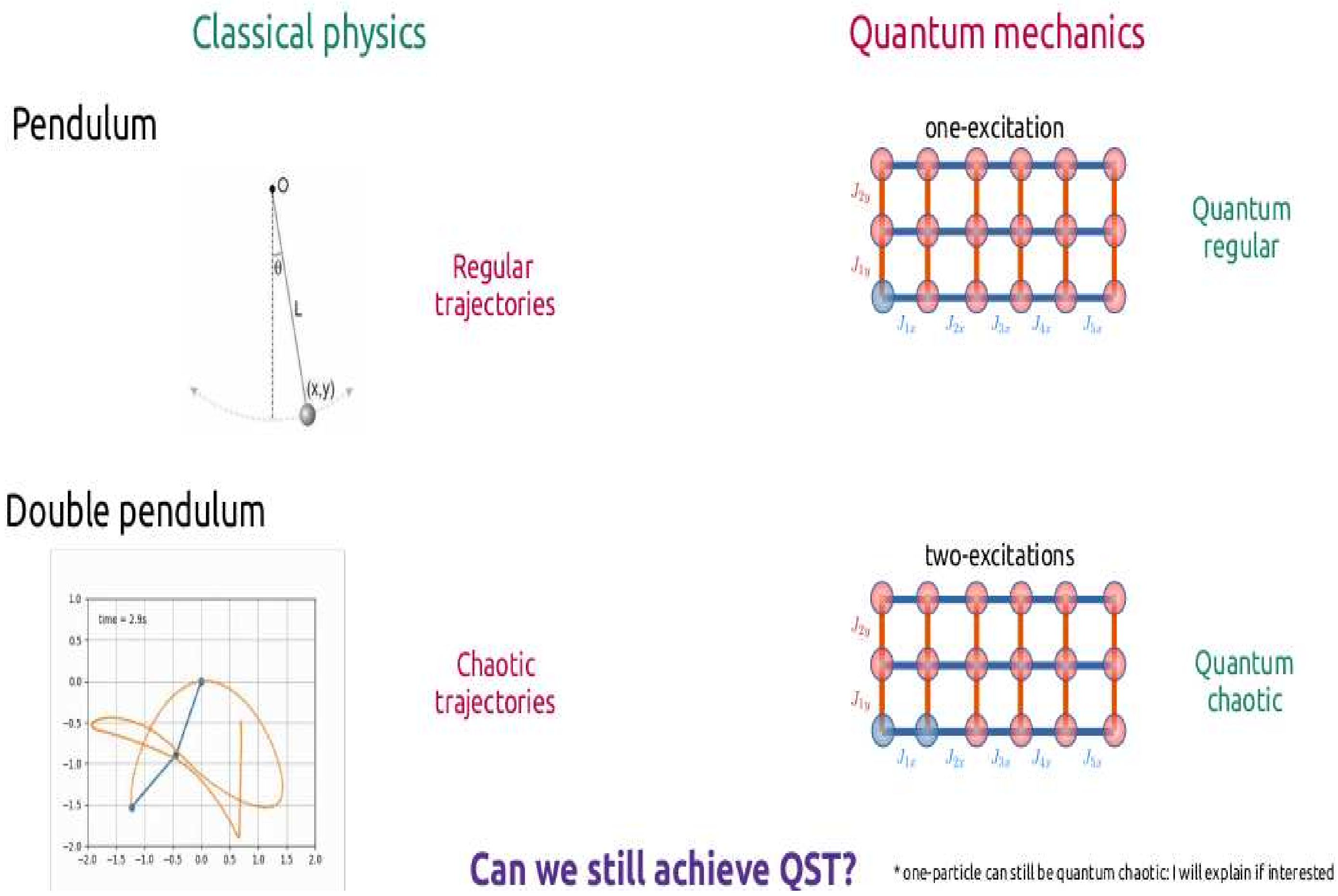
Monte Carlo optimized $\mathcal{H}$ recovers high fidelity (through intricate path).





Preceding: Can compensate for cross-couplings and defective coupler.

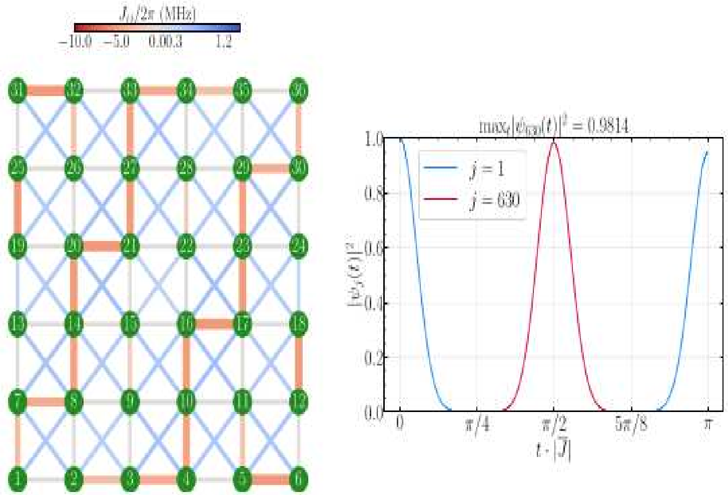
What about multiple excitations?



Can get high fidelity QST in theory (left).

But experimental implementation of theory-guided J_{ij} not quite there yet.

Numerical solutions

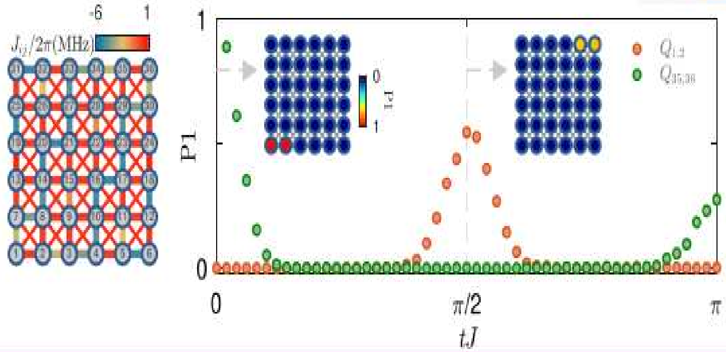


→ Problem is exponentially harder:

$$\mathcal{D}_H = \binom{36}{2} = 630$$

Preliminary experimental results

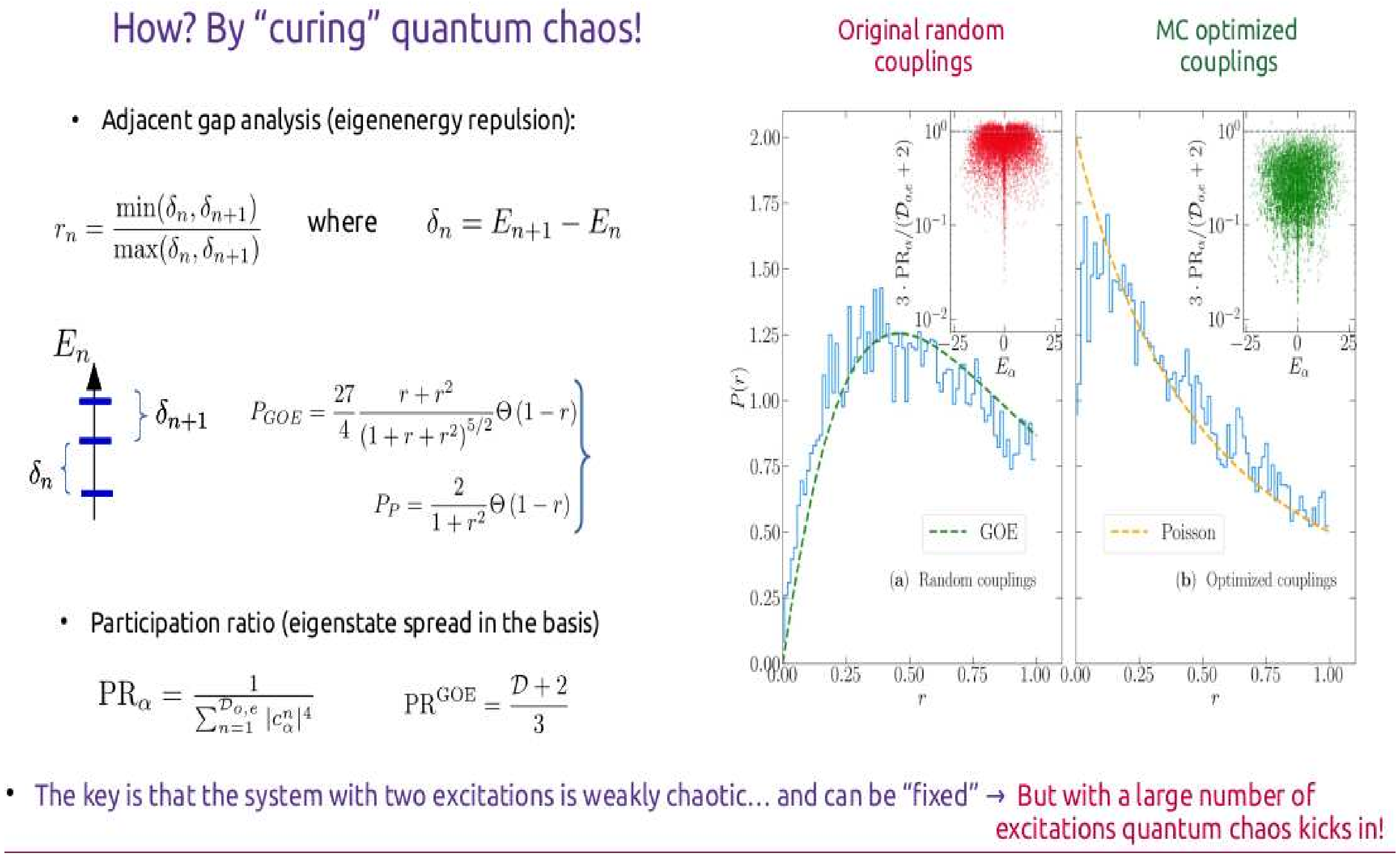
Sensitivity of fine-tuning of the qubit couplings ←



This all seems a black box!

Adjust $\mathcal{H}$ in some (strange) way to get good QST.

Is any insight possible into what's happening?



6. Conclusions

- Usual diffusion of wave function can be circumvented by ‘engineering’.
- Monte Carlo method used in achieving target time evolution operator.
- Generalize Christandl prescription in 1D.
- High fidelity quantum state transfer achievable.

  Cavity-Emitter Arrays (with disorder).

  2D with ‘real world’ effects (cross coupling, dead coupler).

  Multiple excitation (physical insight into where $\mathcal{H}$ evolves.