Quiz 2, Physics 9C, Winter 2016

General Instructions: This quiz is closed book. Only a calculator is allowed. Please show all your work, and give units for all answers and on all graphs. Credit will only be given for complete solutions. The constant $k$ in Coulomb’s law is $k = 9 \cdot 10^9 \text{Nm}^2/\text{C}^2$.

The cube in the figure has sides of length $L = 12 \text{ cm}$. The electric field, has magnitude $E = 600 \text{ N/C}$ and is parallel to the $xy$ plane at an angle $\theta = 20^\circ$ measured from the $+x$ axis towards the $+y$ axis.

(a) What is the electric flux through each of the six faces?
(b) What is the charge density $\rho$?
(c) Is Gauss’ Law obeyed?
(d) If, instead,

$$\vec{E} = (3x + 5y) \hat{x} + (x - y^2) \hat{y}$$

were not uniform, what would the charge density $\rho$ be?

\[ a) \quad \vec{E} = 600 (\cos 20^\circ \hat{x} + \sin 20^\circ \hat{y}) = 564 \hat{x} + 205 \hat{y} \text{ N/C} \]

Flux through a face is, since $\vec{E} = \text{const}$, given by $\vec{E} \cdot \hat{n} \cdot A = \frac{E \cdot \hat{n} \cdot L^2}{\text{area}}$

$S_1$: $\hat{n} = -\hat{y}$ \quad $\vec{E} \cdot \hat{n} \cdot dA = -205 L^2 = -2.95 \text{ Nm}^2/\text{C}$

$S_2$: $\hat{n} = \hat{z}$ \quad $\vec{E} \cdot \hat{n} \cdot dA = 0$

$S_3$: $\hat{n} = +\hat{y}$ \quad $\vec{E} \cdot \hat{n} \cdot dA = +2.95 \text{ Nm}^2/\text{C}$

$S_4$: $\hat{n} = -\hat{x}$ \quad $\vec{E} \cdot \hat{n} \cdot dA = 0$

$S_5$: $\hat{n} = +\hat{x}$ \quad $\vec{E} \cdot \hat{n} \cdot dA = 564 L^2 = 8.12 \text{ Nm}^2/\text{C}$

$S_6$: $\hat{n} = -\hat{x}$ \quad $\vec{E} \cdot \hat{n} \cdot dA = -8.12 \text{ Nm}^2/\text{C}$

\[ b) \quad \text{since } \vec{E} = \text{const } \exists = \epsilon_0 \hat{z} \cdot \vec{E} = \phi \]

\[ c) \quad \text{Gauss law is obeyed since } Q_{\text{enclosed}} = 0 \text{ and } \phi_{\text{tot}} = 0 \]

\[ d) \quad \text{Now } \rho = \epsilon_0 \hat{z} \cdot \vec{E} = \epsilon_0 \left( 3x + 5y \right) + \frac{\partial}{\partial y} (x - y^2) + \frac{\partial}{\partial z} \phi \]

\[ = \epsilon_0 (3 - 2y) \]