[1] a) forward  b) decrease  c) False  d) both  e) $10^{-9}$  f) $p_y$  g) always  h) time reversal  i) none of the above  j) magnitude.

[2] When a force acts on an object, the center of mass of the object will move in the direction of the applied force; hence the × will move to the right in the same direction as the kick. The bottom of the box will move to the right and the top will tend to stay at rest until it gets pulled by the bottom. It will move as shown below. This is the same as a field goal kick or kick off in football. The center of the football moves in the direction of the kick.

[3] a) (8 pts) Method (1) Use conservation of energy: $E_\text{f} = E_\text{i} = \frac{1}{2}mv^2 + mgh = \frac{1}{2}mv'^2$ where $v'$ is the same for all three cases, so all three hit the ground with the same speed. Method (2) Use kinematics: $v_x^2 = v_x^2 + 2g\Delta y = v_y^2 + 2gh$ where the net $y$ displacement is $h$. Knowing that $v'^2 = v_x^2 + v_y^2$ and $v_{ox}$ is constant $\Rightarrow v' = \sqrt{v_x^2 + v_y^2 + 2gh}$ which again is the same for all three cases. b) (8 pts) Using kinematics: Put the origin at the top and down positive $y$ direction.

\[ x = vt \]
\[ h = \frac{1}{2}gt^2, \text{ so } t = \sqrt{\frac{2h}{g}} \text{ and the range is } x = v \sqrt{\frac{2h}{g}}. \]

[4] a) (4 pts) $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow E = \frac{1}{2}(5 \text{ kg})(1.5 \text{ m/s})^2 + \frac{1}{2}(2000 \text{ N/m})(0.80 \text{ m})^2 = 646 \text{ J}$
b) (4 pts) At the max displacement, $A$, all of the energy is potential: $E = \frac{1}{2}kA^2 \Rightarrow$
\[ A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(646 \text{ J})}{2000 \text{ N/m}}} = 0.803 \text{ m} \]
c) (4 pts) At max $v$, $E$ is all kinetic: $E = \frac{1}{2}mv^2 \Rightarrow$
\[ v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(646 \text{ J})}{5 \text{ kg}}} = 16.1 \text{ m/s} \]
d) (4 pts) $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}} = 0.31 \text{ s}.$
e) (4 pts) At $t = 0$ we are close to the max displacement with little velocity so the position vs time curve looks like a cosine curve shifted slightly to the right, so we have a positive initial velocity. $v = dx/dt$, so the velocity looks like a negative sine curve.

[5] a) (10 pts) The initial energy is translational kinetic energy of the center of mass and rotational energy. The final energy is all potential $\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I \omega^2 = mgh$. Every thing is the same for the two objects except for $I$. The object with the largest moment of inertia will roll to the higher point because it has the larger initial energy. b) (10 pts) From part (a) and using $\omega = \frac{v}{r} \Rightarrow$
\[ \frac{1}{2}mv^2 + \frac{1}{2}I \omega^2 = mgh \Rightarrow h = \frac{mv^2}{g}. \]

[6] a) (8 pts) The rain is making an inelastic collision with the train car, so we can only use conservation of momentum in the $x$-direction. $Mv = (M + m_r)v' \Rightarrow$
\[ v' = \frac{Mv}{M + m_r} = \frac{\left(4 \times 10^4 \text{ kg}\right)(10 \text{ m/s})}{4.5 \times 10^5 \text{ kg}} = 8.9 \text{ m/s}. \]
b) (4 pts) The water leaves the car traveling at the
same speed as the car \( \Rightarrow (M + m_r)v' = Mv'' + m_rv' \); hence, \( v'' = v' = 8.9 \text{ m/s} \)
c) (8 pts) From Newton's second law: \( F = \frac{\Delta p}{\Delta t} = \frac{\Delta mv}{\Delta t} \) where \( \frac{\Delta m}{\Delta t} = 30 \text{ kg/s} \) \( \Rightarrow F = (30 \text{ kg/s})(8.9 \text{ m/s}) = 267 \text{ N}. \)

[7] a) (10 pts) The acceleration has two parts. The centripetal acceleration is pointing down toward the center of the Ferris wheel. \( \ddot{a}_c = -r\omega^2 \hat{j} = -(14 \text{ m})(0.4/s^2)\hat{i} = -5.6 \text{ m/s}^2. \) And the tangential acceleration is pointing to the left and is responsible for the increase in \( \omega \) \( \Rightarrow \)

\[
\ddot{a}_t = -\frac{dv}{dt} = r \frac{d\omega}{dt} = -(14 \text{ m})(0.4/s^2)\hat{i} = 5.6 \text{ m/s}^2.
\]

\[
|\ddot{a}| = \sqrt{a^2_t + a^2_c} = \sqrt{(3.5)^2 + (5.6)^2} = 6.6 \text{ m/s}^2
\]

\[
\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{-3.5}{-5.6}\right) = 32^\circ, \text{ i.e. pointing left & 32^\circ below horizontal.}
\]

b) (10 pts) Draw a free–body diagram and sum the forces in the vertical direction and don't forget the centripetal acceleration. \( \sum F_y = N - mg = -ma_c \) \( \Rightarrow N = mg - ma_c \)

\( N = (60 \text{ kg})(9.8 - 3.5) \text{ m/s}^2 \Rightarrow 378 \text{ N}. \) Note if the Ferris wheel is at rest, \( N = 60(9.8) \text{ N} = 588 \text{ N}, \) so the person feels lighter over the top when it is rotating.

[8] Draw a free–body diagram for the strut. Sum the torques about the pivot to eliminate the pivot torques. The lever arm for \( T \) is: \( L = \frac{L}{2} \sin 53.1^\circ, \) for \( w: \frac{L}{2} \)

\[
\text{for } W: L. \quad \sum \tau = \left(\frac{L}{2}\sin 53.1^\circ\right)T - \frac{L}{2}w - LW = 0 \quad \text{Solving for } T:
\]

\[
T = \frac{w + 2W}{4} = \frac{5(10\text{ kg})g + 2(5)(25\text{ kg})g}{4} = 75 \text{ kg}(9.8 \text{ m/s}^2) = 735 \text{ N}.
\]

[9] a) (5 pts) \( L_o = r_o p_o \sin \theta, \) but \( r_o \sin \theta = \ell_o. \) \( \Rightarrow L_o = m v_o \ell_o. \) Similarly at \( \ell_o/3:
\]

\( L = m v_o \ell_o/3 \)

b) (5 pts) No, there is a net torque due to the tension because it doesn't point to the center of the peg. You can see from part (a) that \( L \) is not constant.

c) (5 pts) There is no source of energy and no friction so energy must be conserved. The tension does no work because it is perpendicular to the velocity so \( dW = T \cdot dr = 0. \)

d) (5 pts) Conservation of energy: \( E = E_o \) \( \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 \) \( \Rightarrow v = v_o. \)

[10] a) (5 pts) \( E_o = \frac{1}{2}mr^2 \omega_o^2 - \frac{GMm}{r_o}. \) He is in a circular orbit so there is no radial velocity. \( \Rightarrow E_o = \frac{1}{2}mr^2 \omega_o^2 - \frac{GMm}{r_o}, \) \( L = \ell \omega = mr^2 \omega \) \( \Rightarrow
\]

\[
\omega_o = \frac{L}{mr_o^2} \Rightarrow E_o = \frac{L^2}{2mr_o^2} - \frac{GMm}{r_o}.
\]

b) (5 pts) The effective potential

\[
U_{eff} = \frac{L_o^2}{2mr_o^2} - \frac{GMm}{r_o}
\]

which has a hard core that depends on the angular momentum and an attractive tail which is the \(-GMm/r \).

c) (5 pts) This change puts him into an elliptical orbit with perigee \( r_o \) and apogee \( r_f \).

d) (5 pts) from part (a) the radial velocity is no longer zero so \( E = \frac{1}{2}mr^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}. \)