**General Instructions/Information:** This exam is closed book. Only a calculator is allowed. Please show all your work, and give units for all answers and on all graphs. Credit will only be given for complete solutions. The acceleration of gravity is $g=9.8 \text{ m/s}^2$ downwards.

[1.] An object of mass 7 kg is pulled by a rope with tension 80 N acting at an angle of 15 degrees from the horizontal. (See figure.) The coefficient of kinetic friction between the mass and the table on which it slides is $\mu_k = 0.20$. The object is pulled a distance 7 meters.

a. What is the work done by the tension? b. What is the work done by friction? c. What is the work done by the normal force? d. If the object starts at $v_0 = 6 \text{ m/s}$, what is its final velocity?

\[ T = 80 \text{ N} \]

\[ \theta = 15^\circ \]

\[ d = 7 \text{ m} \]

\[ W_T = \frac{|T| |d| \cos \theta}{|T|} = 80(7) \cos 15 = 540.9 \text{ J} \]

b) \[ f_{\text{fric}} = N \mu_k \]

\[ 80 \sin 15 + N - 7(9.8) = m a_7 = 0 \]

\[ N = 7(9.8) - 80 \sin 15 = 68.6 - 20.7 = 47.9 \text{ N} \]

\[ f_{\text{fric}} = N \mu_k = (47.9)(0.2) = 9.58 \text{ N} \]

\[ W_{\text{fric}} = (9.58)(7) \cos 180 = -67.1 \text{ J} \]

c) \[ W_N = 0 \text{ since } \vec{F} \rightarrow \vec{F} \cos \theta/2 = 0 \]

d) \[ \Delta KE = W_{\text{tot}} \]

\[ \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 540.9 - 67.1 = 473.8 \]

\[ \frac{1}{2} m (6)^2 \]

\[ 126 \text{ J} \]

\[ \frac{1}{2} m 7 (v_f)^2 = 599.8 \]

\[ v_f = 13.1 \text{ m/s} \]
An object of mass $m = 2$ kg moves in the $x$ direction with the potential energy $U(x)$ shown. There are no other forces acting. On the axes provided below the graph of $U(x)$, sketch the force $F(x)$ that the mass experiences. If the object is released from rest at $x = -2$ where $U = 10 J$, what is its velocity at $x = 0$ where $U = -5 J$?

\[ F(x) = -\frac{dU}{dx} \]

\[
\begin{align*}
\text{initial} & \\
K + U & = K + U \\
0 + 10 & = \frac{1}{2} m v^2 + (-5)
\end{align*}
\]

\[
\begin{align*}
15 & = \frac{1}{2} \times 2 \times v^2 + (-5) \\
v & = \sqrt{15} = 3.87 \text{ m/s}
\end{align*}
\]
3. A 0.25 kg glider is moving to the right on a frictionless, horizontal air track with a speed of 1.1 m/s. It has a head-on collision with a 0.40 kg glider that is moving to the left with a speed of 1.5 m/s. The collision is elastic. Write down the equations for conservation of momentum and kinetic energy. Find the velocities (magnitude and direction) of each glider after they collide.

\[ P_{\text{cons}} = (0.25)(1.1) + (0.40)(-1.5) = 0.25V_{1f} + 0.40V_{2f} \]

\[-1.325 = 0.25V_{1f} + 0.40V_{2f} \]

\[ V_{1f} = -1.3 - 1.6V_{2f} \]

\[ E_{\text{cons}} = \frac{1}{2}(0.25)(1.1)^2 + \frac{1}{2}(0.40)(-1.5)^2 = \frac{1}{2}(0.25)V_{1f}^2 + \frac{1}{2}(0.40)V_{2f}^2 \]

\[ 0.15125 + 0.45000 = \frac{1}{8}V_{1f}^2 + \frac{1}{5}V_{2f}^2 \]

\[ = \frac{1}{5}(-1.3 - 1.6V_{2f})^2 + \frac{1}{5}V_{2f}^2 \]

\[ 0.60125 = 1.52V_{2f}^2 + 1.52V_{2f} + 1.2125 \]

\[ 0 = 1.52V_{2f}^2 + 1.52V_{2f} - 0.39 \]

\[ 0 = V_{2f}^2 + V_{2f} - 0.3 \]

\[ 0 = (V_{2f} + 1.3)(2V_{2f} - 1) \]

Solve 1: \[ V_{2f} = -1.5 \quad V_{1f} = 1.1 \quad \text{masses pass through each other w/o colliding!} \]

Solve 2: \[ V_{2f} = 0.15 \quad V_{1f} = -2.1 \quad \text{masses pass through each other w/o colliding!} \]
[4.] On a very muddy football field, a 100 kg linebacker tackles a 80 kg halfback. Immediately before the collision, the linebacker is slipping with a velocity 6 m/s north and the halfback is sliding with a velocity 7 m/s east. What is the velocity (magnitude and direction) at which the two players move together immediately after the collision?

\[
\begin{align*}
\gamma: & \quad 100(0) + 80(0) = 180 \, v_\gamma \\
\chi: & \quad 100(6) + 80(7) = 180 \, v_\chi \\
\end{align*}
\]

\[
\begin{align*}
v_\gamma &= 3.33 \, \text{m/s} \\
v_\chi &= 3.11 \, \text{m/s} \\
|\mathbf{v}| &= 4.56 \, \text{m/s} \\
\theta &= \tan^{-1} \left( \frac{3.33}{3.11} \right) = 47^\circ
\end{align*}
\]