Assignment Five

Due Wednesday, December 1.

[1.] Solve the four site Heisenberg Hamiltonian,

\[ \hat{H} = -J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_4 + \vec{S}_4 \cdot \vec{S}_1) \]

Here \( \hat{S}_{Ix}, \hat{S}_{Iy} \) and \( \hat{S}_{Iz} \) are spin-1/2 operators. As discussed in class, the Hilbert space is 16 dimensional, but using the fact the \([\hat{H}, \hat{S}_{\text{tot}}^z] = 0\), it factorizes into subspaces of dimensions 1, 4, 6, 4, and 1. You may need to diagonalize the six dimensional matrix numerically.

[2.] Optional: This is probably a good warm-up for problem 3, but if you like, jump directly to the Heisenberg model (problem 3).

Write a quantum monte carlo code for

\[ H = -t\hat{\sigma}_x - h\hat{\sigma}_z. \]

Compute \( \langle \hat{\sigma}_z \rangle \) and \( \langle \hat{\sigma}_x \rangle \) and compare against the exact solution.

[3.] Write a world–line quantum monte carlo program for the one-dimensional, spin–1/2 Heisenberg model,

\[ \hat{H} = -J \sum_{i=1}^{N} \vec{S}_i \cdot \vec{S}_{i+1}. \]

As discussed in class, a defect of world–line codes is that they are difficult to check exactly due to the various conservation laws like winding number. I’ll provide some numbers to check against in a few days.