Physics 241– Quantum Magnetism

Problem Set 3 Due 1/27/03

[1.] Consider the four site Heisenberg model, with sites numbered 1, 2, 3, 4 as one goes around the perimeter of the square (see picture). Let's define the state $|1\rangle$ to be the direct product of a singlet on the bond connecting sites 1 and 2 and another singlet on the bond connecting sites 3 and 4. That is,

$$|1\rangle = 1/\sqrt{2}\ [\ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\ ]_{12}\ \text{x}\ 1/\sqrt{2}\ [\ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\ ]_{34}.$$

The subscripts 12 and 34 tell you which sites are being described by the spins inside the brackets. Multiplying out, then,

$$|1\rangle = 1/2\ [\ |\uparrow\downarrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle\ ]$$

Let's define the state $|2\rangle$ to be the direct product of a singlet on the bond connecting sites 1 and 4 and another singlet on the bond connecting sites 2 and 3. (Work out what this is in analogy with the above construction.) Compute the lowest possible energy of a "resonating valence bond" (RVB) trial wavefunction for the four site Heisenberg model.

$$|\Psi_{\text{RVB}}\rangle = \alpha|1\rangle + \beta|2\rangle$$

by minimizing

$$\langle\Psi_{\text{RVB}}|H|\Psi_{\text{RVB}}\rangle.$$

Compare with the true ground state you found in problem 1 of homework assignment two. In the late 1980's, Anderson proposed that this sort of state (or a variant thereof) formed the ground state of the Heisenberg model, as opposed to a state with long range antiferromagnetic magnetic order.