Consider the four site Heisenberg model, with sites numbered 1, 2, 3, 4 as one goes around the perimeter of the square (see picture). Let’s define the state $|1\rangle$ to be the direct product of a singlet on the bond connecting sites 1 and 2 and another singlet on the bond connecting sites 3 and 4. That is,

$$|1\rangle = \frac{1}{\sqrt{2}} \left[ |↑↓⟩ - |↓↑⟩ \right]_{12} \times \frac{1}{\sqrt{2}} \left[ |↑↓⟩ - |↓↑⟩ \right]_{34}.$$  

The subscripts 12 and 34 tell you which sites are being described by the spins inside the brackets. Multiplying out, then,

$$|1\rangle = \frac{1}{2} \left[ |↑↓↑↓⟩ - |↑↓↓↑⟩ - |↓↑↑↓⟩ + |↓↑↓↑⟩ \right].$$

Let’s define the state $|2\rangle$ to be the direct product of a singlet on the bond connecting sites 1 and 4 and another singlet on the bond connecting sites 2 and 3. (Work out what this is in analogy with the above construction.) Compute the lowest possible energy of a “resonating valence bond” (RVB) trial wavefunction for the four site Heisenberg model.

$$|Ψ_{RVB}\rangle = α|1\rangle + β|2\rangle$$

by minimizing

$$\langle Ψ_{RVB}|H|Ψ_{RVB}\rangle.$$  

Compare with the true ground state you found in problem 1 of homework assignment two. In the late 1980's, Anderson proposed that this sort of state (or a variant thereof) formed the ground state of the Heisenberg model, as opposed to a state with long range antiferromagnetic magnetic order.