Physics 241– Quantum Magnetism

Problem Set 2 Due 1/22/03

[1.] Use the trick described in class to get the sixteen eigenvalues of $H = J\ [\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_4 + \vec{S}_4 \cdot \vec{S}_1]$ (the 'four site Heisenberg model'). Can you construct the eigenvectors?

[2.] Show that the 'fluctuation form' for the magnetic susceptibility, $\chi = \beta[\langle \mu_z^2 \rangle - \langle \mu_z \rangle^2]$ gives the same result as the expression used in class, $\langle \mu_z \rangle = \mu_{\rm B} \tanh(\beta \mu_{\rm B} B)$, and hence $\chi = d\langle \mu_z \rangle / dB = \beta\, \mu_{\rm B}^2\, {\rm sech}^2(\beta \mu_{\rm B} B)$, for a single spin–1/2 in a field $\vec{B} = B\hat{z}$ with $E = -\mu_{\rm B} 2 s_z B$ and $s_z = \pm 1/2$.

[3.] In class we derived the ratio $\mu_{\rm eff}/\mu_{\rm B} = 2.54$ for Ce^{3+} using Hund's rules and the Lande g-factor, and noted it was very close to the experimental result $\mu_{\rm expt}/\mu_{\rm B} = 2.3$. Repeat the calculation of $\mu_{\rm eff}/\mu_{\rm B}$ for Pr^{3+} and compare to $\mu_{\rm expt}/\mu_{\rm B} = 3.4$.

[4.] [Optional (but highly encouraged!) Numerical Problem] Set up the 16x16 matrix of the Hamiltonian in problem 1 and diagonalize it numerically using the routines and instructions given on the course web-site (http://leopard.ucdavis.edu/rts/p241/physics241.html). Note that there are far fewer than 256 matrix elements to enter, so it is not too tedious to assign them all individually in your program. However, to prepare you for later assignments, you might want to start thinking about ways to get the computer to generate the matrix for you.