Physics 241– Quantum Magnetism

Problem Set 1 Due 1/13/03

[1.] In class we wrote down two different bases for the four dimensional Hilbert space of two spin-1/2 objects. Construct the (4x4) matrix of the operator $J\,\vec{S}_1 \cdot \vec{S}_2$ in the basis $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$. Here J is just some number. Diagonalize this matrix and compute the eigenvectors and eigenvalues. How are the eigenvectors related to the second basis we wrote for the Hilbert space whose vectors had well defined symmetry properties under the exchange of the two spin labels? Assuming $J > 0$, do the two spins point parallel or antiparallel to each other in the eigenvector of lowest eigenvalue ?

Use the fact that $\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}[(\vec{S}_1 + \vec{S}_2)^2 - S_1^2 - S_2^2]$ and the rules for adding angular momentum to reproduce your result for the eigenvalues without actually diagonalizing.

As we will see in a week or two, this problem is actually the Heisenberg model on a very small (two site!) lattice. J is the exchange energy.

[2.] Use Hund's rules to determine the ground state S, L, J of N, O, Fl, and Ne. What are the spectroscopic designations of the ground states?