[1.] Construct the matrix which will reflect \( \mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} \) about a line through the origin making an angle \( \theta \) with the \( x \)-axis.

[2.] Show that the only allowed rotation angles for a 2D Bravais lattice are \( \theta = \pi, \ 2\pi/3, \ \pi/2, \) and \( \pi/3 \). Hint: Write the two lattice vectors

\[
\mathbf{a}_1 = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad \mathbf{a}_2 = \begin{pmatrix} b \\ c \end{pmatrix}
\]

and demand that when \( \mathbf{a}_1 \) is rotated by \( \theta \) and by \( -\theta \) that it lands on another site \( n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 \) of the Bravais lattice.

[3.] The “family” of high temperature (cuprate) superconductors contains many compounds. \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) is one example. What is the lattice structure of the “parent” compound, \( \text{La}_2\text{CuO}_4 \)? What is a “perovskite”? \( \text{La}_2\text{CuO}_4 \) is an antiferromagnetic insulator. When \( \text{Sr} \) replaces some of the \( \text{La} \), one produces a high temperature superconductor.

[4.] Find the primitive lattice vectors \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) of the real space graphene (hexagonal lattice) and also \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) for the reciprocal space lattice. How long are \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) if the C-C distance is \( a \sim 0.142 \text{ nm} \)?

[5.] Sketch the Bravais lattice, find \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) and draw the unit cell for the \( \text{CuO}_2 \) lattice in Fig. 1(left). Do the same for a lattice in which the oxygen atoms are inequivalent, Fig. 1(right), as is the case in cuprate superconductors when the oxygen tetrahedra tilt. (See problem 3.).
[6.] Find the volumes of the unit cells for bcc and fcc lattices.

[7.] (a) Find $c/a$ for an hcp lattice. (b) At temperature $23^\circ$, sodium changes from bcc to hcp. If the number density is unchanged, find $a$ for the high temperature hcp phase if the low temperature bcc phase has $a' = 4.23 \, \text{Å}$. 

[8.] In a perfect crystal the atoms are located at positions $\mathbf{R}_n = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$. Diffraction peaks are $\delta$-functions at reciprocal lattice vectors $\mathbf{G} = m_1 \mathbf{b}_1 + m_2 \mathbf{b}_2 + m_3 \mathbf{b}_3$. Suppose now some “imperfections” are present so that $\mathbf{R}_n = \mathbf{R}^0_n + \mathbf{S}_n$ where each component of $\mathbf{S}_n$ has a Gaussian distribution $P(S_i) = \exp(-S^2_i/(2S^2_0))/\sqrt{2\pi S_0}$. How are the intensities of the diffraction peaks changed from their perfect delta-function forms?