[1.] Verify that
\[ \int_{(0,0)}^{(2,1)} z^* \, dz \]
depends on the path by evaluating the integral for the path which consists of moving horizontally from (0, 0) to (2, 0) and then vertically from (2, 0) to (2, 1), and the path which consists of moving vertically (0, 0) to (0, 1) and then horizontally from (0, 1) to (2, 1). By Cauchy’s integral theorem, shouldn’t the two paths give equal results? What went wrong?

[2.] Evaluate the integral
\[ \int_C (z - z_0)^n \, dz \]
where \( C \) is a contour which consists of a circle of radius \( R \) about the point \( z_0 \).

[3.] From the definition of the Laplace transform, show that the transform of \( f(x) = e^{-ax} \) is \( F(s) = \frac{1}{s+a} \). Find the inverse Laplace transform of \( \frac{1}{(s+a)(s+b)} \) for \( a \neq b \).

[4.] The diffusion equation,
\[ \frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2} \]
describes the evolution of the density \( \rho(x, t) \). Suppose you discretize time in steps of \( dt \) and space into a mesh of size \( dx \), so that \( \rho(n, k) \) gives the density \( \rho \) at spatial location \( x = n \, dx \) and time \( t = k \, dt \). Derive an expression for \( \rho(m, k + 1) \) in terms of \( \rho(n, k) \). That is, write the discretized version of the diffusion equation.

[5.] Prove that matrix multiplication is associative. That is, demonstrate \( A(BC) = (AB)C \) if \( A, B, \) and \( C \) are \( N \times N \) matrices.

[6.] A linear quantum oscillator in its ground state has a wavefunction
\[ \psi(x) = a^{-1/2} \pi^{-1/4} e^{-x^2/2a^2}. \]
What is the corresponding momentum wave function \( \phi(p) \)?

BONUS QUESTION! \( \psi(x) \) is normalized so that \( \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1 \). You notice that the momentum wave function is also normalized to unity. Is this an accident? Can you prove that for any normalized \( \psi(x) \) the corresponding \( \phi(p) \) will be normalized?