"Translation operators"

Take a step back from angular momentum and put it together with earlier things learned (115A)

$$e^{-i\hat{H}t/\hbar} : \quad \text{"time evolution operator"}$$

$\downarrow$

aka "time translation operator" why?

$$e^{-i\hat{H}t/\hbar}|\psi(t_0)\rangle = |\psi(t_0+t)\rangle$$

translates you forward in time

Consider now $e^{i\hat{p}a/\hbar}$ work in $|x\rangle$ basis where we have convenient form for $\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$

$$e^{i\hat{p}a/\hbar} = e^{a\,d/dx}$$

$$e^{i\hat{p}a/\hbar}\psi(x) = e^{a\,d/dx}\psi(x) = \left[1 + a\frac{d}{dx} + \frac{1}{2}a^2\frac{d^2}{dx^2} + \ldots\right]\psi(x)$$

$$= \psi(x) + a\psi'(x) + \frac{1}{2}a^2\psi''(x) + \ldots$$

$$e^{i\hat{p}a/\hbar}\psi(x) = \psi(x+a)$$

translates you spatially $x \to x+a$

$$e^{iL_z\alpha/\hbar} f(r,\theta,\phi) = f(r,\theta,\phi+\alpha)$$

~~rotates about $\hat{z}$ axis $\phi \to \phi+\alpha$~~

In general $e^{i\vec{L}\cdot\hat{n}\alpha}$ rotates by α about axis $\hat{n}$

$\hat{H}$ is generator of time translations

$\vec{p}$ is generator of spatial translations

$\vec{L}$ is generator of rotations

group theory / generators etc

generators commute, do not commute $\begin{cases} \text{Abelian} \\ \text{non Abelian} \end{cases}$

$O(3)$ $SU(2)$ $SU(3)$

quarks

Why worry about group theory?!

embodies symmetry in nature

group of 3×3 orthogonal matrices

group of 2×2 unitary matrices of determinant +1

$SU(3) \leftarrow 3\times 3$ unitary matrices of det $= 1$

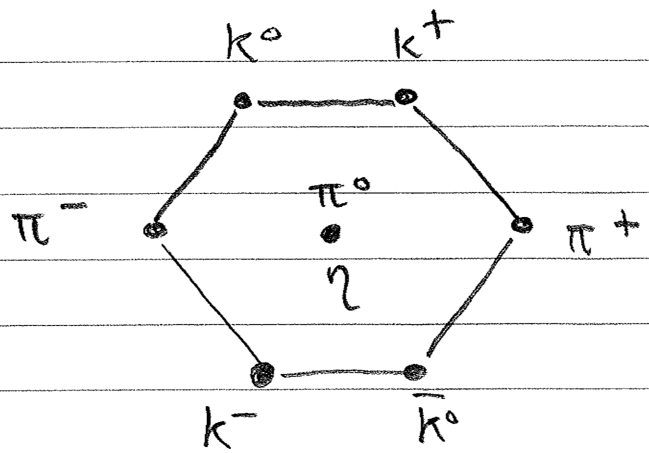
3×3 complex matrices $\rightarrow$ 18 parameters

unitary: $\begin{bmatrix} \alpha + i\overset{\nearrow\phi}{\beta} & \gamma + i\delta & \\ \gamma - i\delta & & \\ & & \end{bmatrix}$

diagonal real
$18 - 3 = 15$
offdiagonal are complex conjugates
$15 - 6 = 9$
det $= +1$ $\quad 9 - 1 = 8$

"The eight fold way"

Meson octet



Baryon octet