

3D SHO ← 1

Another Degenerate PT Example is 3D SHO

$$|n_x n_y n_z\rangle$$

$$E_{n_x n_y n_z} = (n_x + n_y + n_z + \frac{3}{2}) \hbar \omega$$

$$H = \hbar \omega_x (a_x^\dagger a_x + \frac{1}{2})$$

$$+ \hbar \omega_y (a_y^\dagger a_y + \frac{1}{2})$$

$$+ \hbar \omega_z (a_z^\dagger a_z + \frac{1}{2})$$

$$\phi_{n_x n_y n_z}(x, y, z) = \left(\frac{m\omega_x}{\hbar}\right)^{1/4} \frac{1}{[2^{n_x} n_x!]}^{1/2} H_{n_x} \left(\sqrt{\frac{m\omega_x}{\hbar}} x\right) e^{-\frac{m\omega_x x^2}{\hbar}}$$

... (y) ...

... (z) ...

Perturbation  $Bxy = H'$

eg  $V(x, y, z) = \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2 + \frac{1}{2} m\omega_z^2 z^2$

+ Bxy

↗

H'

Why use raising/lowering ops?

(1) Computational simplicity

$\langle x^2 \rangle$  in state  $\psi_n(x)$        $x = \sqrt{\frac{\hbar}{2m\omega}} (a + a^\dagger)$

(a)  $\langle n | \frac{\hbar}{2m\omega} (a + a^\dagger)^2 | n \rangle$

$= \frac{\hbar}{2m\omega} \langle n | a a + a a^\dagger + a^\dagger a + a^\dagger a^\dagger | n \rangle = (2n+1) \frac{\hbar}{2m\omega}$

vs (b)  $\int \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \frac{1}{2^n n!} H_n^2 \left(\sqrt{\frac{m\omega}{\hbar}} x\right) x^2 e^{-2m\omega x^2/\hbar} dx$

(2) Better way to do QM especially relativistic QM where particles can be created and destroyed

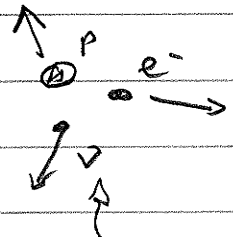
neutron decay  $\leftarrow$  how long does it live 881 seconds

$\psi(x, y, z, t)$

$\downarrow ??$

$\psi(x_p, y_p, z_p, x_e, y_e, z_e, x_\nu, y_\nu, z_\nu, t)$

$\textcircled{n}$



how do we know?

Suddenly function has 9 variables instead of 3 ?!

What's in the news right now about  $\nu$

Physicists use creation / destruction operators to  $\downarrow$  QM  
 where # particles can change, Schematically

$a_{\text{neutrino}}^\dagger$   $a_{\text{electron}}^\dagger$   $a_{\text{proton}}^\dagger$   $a_{\text{neutron}}$   $\leftarrow$  piece of H

$a_{\text{neutrino}, \vec{k}}^\dagger$   $a_{\text{electron}, \vec{q}}^\dagger$   $a_{\text{proton}, \vec{p}}^\dagger$   $a_{\text{neutron}, \vec{p}_0}$   $\leftarrow$  more precise

Guess about  $\vec{k}, \vec{p}, \vec{q}, \vec{p}_0$  ?

$$\vec{k} + \vec{p} + \vec{q} = \vec{p}_0$$

3PSHO-2

Set  $\omega_x \neq \omega_y \neq \omega_z$

First order shift

$$\langle n_x n_y n_z | B \sqrt{\frac{\hbar}{2m\omega_x}} \sqrt{\frac{\hbar}{2m\omega_y}} (a_x^\dagger + a_x)(a_y^\dagger + a_y) | n_x n_y n_z \rangle = 0$$

Second order shift, what intermediate states  $|m_x m_y m_z\rangle$  are "connected" to  $|n_x n_y n_z\rangle$  by  $H'$ ?

- ①  $|n_x+1, n_y+1, n_z\rangle$   $\sqrt{n_x+1} \sqrt{n_y+1}$
- ②  $|n_x-1, n_y+1, n_z\rangle$   $\sqrt{n_x} \sqrt{n_y+1}$
- $|n_x+1, n_y-1, n_z\rangle$   $\sqrt{n_x+1} \sqrt{n_y}$
- $|n_x-1, n_y-1, n_z\rangle$   $\sqrt{n_x} \sqrt{n_y}$

Square of ME  $\rightarrow B^2 \frac{\hbar}{2m\omega_x} \frac{\hbar}{2m\omega_y}$

Eg for ①  $\frac{(n_x+1)(n_y+1) B^2 (\hbar/2m\omega_x) (\hbar/2m\omega_y)}{-\hbar\omega_x - \hbar\omega_y}$

②  $\frac{n_x(n_y+1) B^2 (\hbar/2m\omega_x) (\hbar/2m\omega_y)}{+\hbar\omega_x - \hbar\omega_y}$

~~Could do degenerate pert.~~

3D-SHO-3

What if degenerate?

In that case there is a first order shift

because

$$|n_x n_y n_z\rangle$$

connected to

$$\langle n_x - 1 \ n_y + 1 \ n_z |$$

by  $\hat{H}'$  and they have same energy