Another Degenerate PT Example is 3D SHO

$$|n_x\, n_y\, n_z\rangle$$

$$H = \hbar\omega_x(a_x^\dagger a_x + 1/2) + \hbar\omega_y(a_y^\dagger a_y + 1/2) + \hbar\omega_z(a_z^\dagger a_z + 1/2)$$

$$E_{n_x n_y n_z} = (n_x + n_y + n_z + \tfrac{3}{2})\hbar\omega$$

$$\phi_{n_x n_y n_z}(x,y,z) = \left(\frac{m\omega_x}{\pi\hbar}\right)^{1/4} \frac{1}{[2^{n_x} n_x!]^{1/2}} H_{n_x}\left(\sqrt{\frac{m\omega_x}{\hbar}}\,x\right) e^{-\frac{m\omega_x x^2}{\hbar}}$$

$$\ldots (y) \ldots$$

$$\ldots (z) \ldots$$

Perturbation $Bxy = H'$

eg $V(x,y,z) = \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2 + \frac{1}{2} m\omega_z^2 z^2$

$$\underbrace{+\,Bxy}_{H'}$$

3DSHO-1A

Why use raising lowering op?

(1) Computational simplicity

$\langle x^2 \rangle$ in state $\psi_n(x)$ $\qquad x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$

(a) $\langle n | \frac{\hbar}{2m\omega}(a+a^\dagger)^2 | n \rangle$

$= \frac{\hbar}{2m\omega} \langle n | \underbrace{aa}_{\phi} + \underbrace{aa^\dagger}_{n+1} + \underbrace{a^\dagger a}_{n} + \underbrace{a^\dagger a^\dagger}_{\phi} | n \rangle = (2n+1)\frac{\hbar}{2m\omega}$

vs (b) $\int \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \frac{1}{2^n n!} H_n^2\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right) x^2 e^{-2m\omega x^2/\hbar}$!

(2) Better way to do QM especially relativistic QM where particles can be created and destroyed

neutron decay ← how long does n live 881 seconds

$\psi(x, y\, z\, t)$ $\qquad$ n → p e⁻ ν

$\downarrow$??

$\psi(x_p\, y_p\, z_p\, x_e\, y_e\, z_e\, x_\nu\, y_\nu\, z_\nu\, t)$

how do we know? What's in the news right now about ν

Suddenly function has 9 variables instead of 3 ?!

3DSU0-1B

relativistic

Physicists use creation/destruction operators to do QM where # particles can change. Schematically

$a^\dagger_{neutrino}\, a^\dagger_{electron}\, a^\dagger_{proton}\, a_{neutron}$ ← piece of H

$a^\dagger_{neutrino,\vec{k}}\, a^\dagger_{electron,\vec{q}}\, a^\dagger_{proton,\vec{p}}\, a_{neutron,\vec{p}_0}$ ← more precise

Guess about $\vec{k}, \vec{p}, \vec{q}, \vec{p}_0$?

$$\vec{k}+\vec{p}+\vec{q} = \vec{p}_0$$

3DSHO-2

Set $\omega_x \neq \omega_y \neq \omega_z$

First order shift

$$\langle n_x\, n_y\, n_z |\, B \sqrt{\frac{\hbar}{2m\omega_x}} \sqrt{\frac{\hbar}{2m\omega_y}} (a_x^\dagger + a_x)(a_y^\dagger + a_y) | n_x n_y n_z \rangle$$

$$= 0$$

Second order shift, What intermediate states $|m_x\, m_y\, m_z\rangle$

are "connected" to $|n_x\, n_y\, n_z\rangle$ by H' ?

1. $|n_x+1 \quad n_y+1 \quad n_z\rangle \qquad \sqrt{n_x+1}\,\sqrt{n_y+1}$
2. $|n_x-1 \quad n_y+1 \quad n_z\rangle \qquad \sqrt{n_x}\,\sqrt{n_y+1}$

$|n_x+1 \quad n_y-1 \quad n_z\rangle \qquad \sqrt{n_x+1}\,\sqrt{n_y}$

$|n_x-1 \quad n_y-1 \quad n_z\rangle \qquad \sqrt{n_x}\,\sqrt{n_y}$

Square of ME $\longrightarrow B^2 \frac{\hbar}{2m\omega_x} \frac{\hbar}{2m\omega_y}$

E_0 for ①
$$\frac{(n_x+1)(n_y+1)\, B^2 (\hbar/2m\omega_x)(\hbar/2m\omega_y)}{-\hbar\omega_x - \hbar\omega_y}$$

②
$$\frac{n_x (n_y+1)\, B^2 (\hbar/2m\omega_x)(\hbar/2m\omega_y)}{+\hbar\omega_x - \hbar\omega_y}$$

~~Could do degenerate P.T.~~

What if degenerate?

In that case there is a <u>first</u> order shift

because $|n_x\ n_y\ n_z\rangle$

connected to $\langle n_x - 1\ \ n_y + 1\ \ n_z|$

by H' and they have <u>same</u> energy