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Richard Scalettar Physics 409
Course web site:
http://leopard.physics.ucdavis.edu/rts/p115b/p115b_F2011.html

{ Mo Fr noon-1 / We 1-2 in 106 } office hours

Course Syllabus

2 MT Exams 15% each
1 Final Exam 20%
Weekly HW 50%

} grading — Dave Cone 401 Th 3-4

115A Griffiths 1,2,3

Chapters to be covered Griffiths 6,4,5,9,11,...

Additional Material

C-programming → numerical solns to QM *

$e^{-\beta\hat{H}} \leftarrow e^{-i\hat{H}t}$ Quantum Stat Mech (Griffiths ch 5) → Path integral SHO

Why start with 6.1,2 (perturbation theory)?
(See Griffiths preface, top p viii)
My reason: opportunity to review Ch 1-3 before new material of 4,5.
6.3,4 need Ch 4: Perturbation theory for H-atom

Q: Could you solve

$$* \quad m\ddot{x} = -kx - \underbrace{ax^3}_{\text{trivial with computer}}$$

HW1 ① Diff. Eqn DE-2
② $\langle E \rangle$ at Temp T PI-SHO-4

Q: We will be doing Hatom (like Kepler for classical)
do numerically easy!

Q: What Extra-solar planet discovery has been in news: "Tatooine"

(1)

Topics covered in 115A

Chapters 1-3 of Griffiths

1) Review of probability concepts
Schroedinger Eqn – meaning of $\Psi(x,t)$, normalization
Momentum operator, current operator,

2) Time independent Schroedinger Eqn
Infinite square well – orthonormality and completeness of ϕ_n
Harmonic oscillator – algebraic method (a, a^+)
– sch. Eqn / series method / Hermite polys

Free particle
Delta function potential
Finite square well

3) Hilbert space
Hermitian operators – eigenfunctions and eigenvalues
$\psi(x) \Leftrightarrow \phi(p)$ Fourier transform
uncertainty principles $\Delta x \Delta p$ $\Delta t \Delta E$
Dirac notation

Ch 6 Time Independent Perturbation Theory

$$H^\circ |\psi_n^\circ\rangle = E_n^\circ |\psi_n^\circ\rangle$$

Hamiltonian <u>operator</u>

nth eigenvalue

nth <u>eigenvector</u> of H°

$$H^\circ \to H^\circ + \lambda H'$$

perturbation

time independent $H' \neq H'(t)$

Q: Can you think of a time dependent H' of interest? Suppose you are studying physics of an atom?

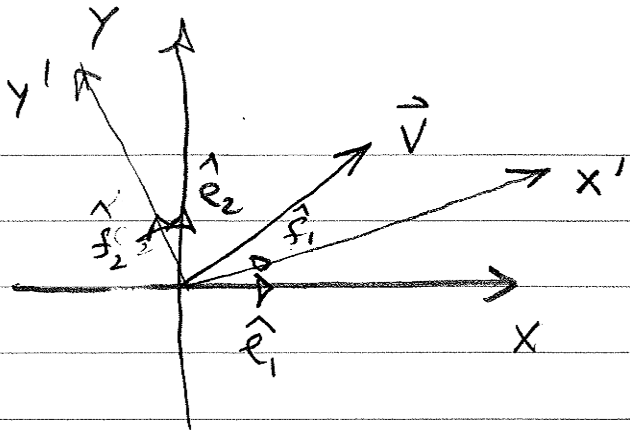
A: light incident on atom $\vec{E}(t), \vec{B}(t)$!

Lets review notation before proceeding (saw this all in 115A probably).

R1

vector $\vec{V}$

basis $\hat{e}_1$ $\hat{e}_2$



$$\vec{V} = v_1 \hat{e}_1 + v_2 \hat{e}_2$$

components $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ ← depend on basis

$\hat{e}_1 \cdot \vec{V} = v_1$ $\quad \hat{e}_2 \cdot \vec{V} = v_2$ $\quad$ If $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$

New basis $\vec{V} = v_1' \hat{f}_1 + v_2' \hat{f}_2$

$$\hat{f}_1 = (\hat{f}_1 \cdot \hat{e}_1)\,\hat{e}_1 + (\hat{f}_1 \cdot \hat{e}_2)\,\hat{e}_2$$

$$\hat{f}_2 = (\hat{f}_2 \cdot \hat{e}_1)\,\hat{e}_1 + (\hat{f}_2 \cdot \hat{e}_2)\,\hat{e}_2$$

$$\vec{V} = \left[(\hat{f}_1 \cdot \hat{e}_1)\, v_1' + (\hat{f}_2 \cdot \hat{e}_1)\, v_2'\right] \hat{e}_1$$

$$+ \left[(\hat{f}_1 \cdot \hat{e}_2)\, v_1' + (\hat{f}_2 \cdot \hat{e}_2)\, v_2'\right] \hat{e}_2$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \hat{f}_1 \cdot \hat{e}_1 & \hat{f}_2 \cdot \hat{e}_1 \\ \hat{f}_1 \cdot \hat{e}_2 & \hat{f}_2 \cdot \hat{e}_2 \end{pmatrix}}_{S} \begin{pmatrix} v_1' \\ v_2' \end{pmatrix}$$

$S \equiv$ "change of basis matrix"

R2

If $\hat{f}_i \cdot \hat{f}_j = \delta_{ij}$ and $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$

"orthonormal" then $S^{-1} = S^T$ $\quad SS^T = I$

$$S^T \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1' \\ v_2' \end{pmatrix} \qquad (v_1 \, v_2) S = (v_1' \, v_2')$$

(reverse order when transposing, $(AB)^T = B^T A^T$)

In new bases

$$|\vec{v}|^2 = {v_1'}^2 + {v_2'}^2$$

$$= (v_1' \, v_2') \begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = (v_1 \, v_2) S \, S^T \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$= (v_1 \, v_2) \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = v_1^2 + v_2^2$$

If you use orthonormal bases length of vector is unchanged when basis altered!

$$M \vec{v} = \vec{w}$$

Matrix (operator) vector new vector

M also depends on basis!!

R3

Key desired feature:

$\vec{u} \cdot M\vec{v}$ same number in all bases

Tells us how M must change when $\{\hat{e}_i\} \to \{\vec{f}_i\}$

New bases $(u_1' \; u_2') M' \begin{pmatrix} v_1' \\ v_2' \end{pmatrix}$

$= (u_1 \; u_2) S M' S^T \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

REQUIRE $= (u_1 \; u_2) M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\therefore \quad S M' S^T = M \qquad M' = S^T M S$

SKIP?

Allows one to speak of the eigenvectors of a matrix:

Suppose $M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

then $S M' S^T S \begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \lambda S \begin{pmatrix} v_1' \\ v_2' \end{pmatrix}$

$S M' \begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = S \lambda \begin{pmatrix} v_1' \\ v_2' \end{pmatrix}$

$M' \begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \lambda \begin{pmatrix} v_1' \\ v_2' \end{pmatrix}$ ← Eigenfunctions in new bases as well!

2.

Back to QM:

Q: What is connection between abstract Hilbert space

$$* \quad \hat{H}^0 |\psi_n^0\rangle = E_n^0 |\psi_n^0\rangle$$

$$** \text{ and } \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi_n^0(x) = E_n^0\,\psi_n^0(x) \quad ?$$

A: In going from * to ** we picked a basis. (what basis? $\hat{X}|x\rangle = x|x\rangle$)

$\therefore$ $\psi_n^0(x)$ are components of vector $|\psi_n^0\rangle$ in this basis and $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$ is how $\hat{H}^0$ looks in this basis

$$\vec{V} \rightarrow \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \qquad V_1 = \hat{e}_1 \cdot \vec{V}$$

vector components

$$|\psi_n^0\rangle \rightarrow \psi_n^0(x)$$

like 1, 2 label except ∞ # of them

$$\psi_n^0(x) = \langle x|\psi_n^0\rangle$$

$$V_1 = (\hat{f}_1 \cdot \hat{e}_1)V_1' + (\hat{f}_2 \cdot \hat{e}_1)V_2' = \sum_i \hat{e}_1 \cdot \hat{f}_i V_i'$$

$$\psi_n^0(x) = \int dp\, \langle x|p\rangle\, \psi_n^0(p)$$

$$\left[\quad \langle x|\psi_n^0\rangle = \int dp\, \langle x|p\rangle\langle p|\psi_n^0\rangle \quad\right]$$

overlaps of $|x\rangle$ and $|p\rangle$ basis vectors

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Does anyone know what these numbers $\langle x|p\rangle$ are?

SKIP: $\psi_n^\circ(x) = \frac{1}{\sqrt{2\pi}}\int e^{ikx}\,\psi_n^\circ(k)\,dk$ Eq 2.100

$p = \hbar k$

DIRECT TO $\longrightarrow$ $\psi_n^\circ(x) = \frac{1}{\sqrt{2\pi\hbar}}\int e^{ipx}\,\psi_n^\circ(p)\,dp$

$$\langle x|p\rangle = \frac{e^{ipx}}{\sqrt{2\pi\hbar}}$$

Words: Components of eigenstate of $\hat{p}$ in bases of eigenstates of $\hat{x}$ are

$$\frac{1}{\sqrt{2\pi\hbar}}\,e^{ipx}$$

corresponding eigenvalues