

Summary of Procedure to
 Add Angular Momentum
 + Further General Discussion

① We never ask how to add linear momentum because

there are no nonzero commutation relations to make anything

but the "obvious" result true: If particle 1 has linear

momentum $\vec{p}_1 = (p_{1x}, p_{1y}, p_{1z})$ and particle 2 has

$\vec{p}_2 = (p_{2x}, p_{2y}, p_{2z})$ then $\vec{p}_{\text{tot}} = \vec{p}_1 + \vec{p}_2 = (p_{1x} + p_{2x}, p_{1y} + p_{2y}, p_{1z} + p_{2z})$

Angular momentum is different because $[L_x, L_y] = i\hbar L_z$

etc!

② Given particle 1 with $L_1^2 = l_1(l_1 + 1)\hbar^2$

and $L_{z1} = m_1\hbar$ ($m_1 = -l_1, -l_1 + 1, \dots, +l_1$)

and particle 2 with $L_2^2 = l_2(l_2 + 1)\hbar^2$

and $L_{z2} = m_2\hbar$ ($m_2 = -l_2, -l_2 + 1, \dots, +l_2$)

then $(L_1 + L_2)^2 = l(l + 1)\hbar^2$ with $l = l_1 + l_2, l_1 + l_2 - 1, \dots$

$|l_1 - l_2|$. For each l , $m = -l, -l + 1, \dots, +l$

③ We saw two examples of this in class, and saw the "counting" worked out, i.e. the dimensions of the Hilbert spaces

using $L_1^2, L_2^2, L_{12}, L_{22}$ as complete set of commuting

operators and $L_1^2, L_2^2, (L_1+L_2)^2, L_{12}+L_{22}$ agreed.

$$l_1 = l_2 = \frac{1}{2} \quad (2l_1 + 1) = 2 \quad (2l_2 + 1) = 2$$

$$2 \times 2 = 4 \text{ dim Hilbert space}$$

$$\left. \begin{array}{l} l = 1 \quad (2l + 1) = 3 \\ \quad 0 \quad (2l + 1) = 1 \end{array} \right\} \begin{array}{l} \nearrow \checkmark \\ 4 \text{ dim Hilbert space} \end{array}$$

$$l_1 = l_2 = 1 \quad (2l_1 + 1) = 3 \quad (2l_2 + 1) = 3$$

$$3 \times 3 = 9 \text{ dim Hilbert space}$$

$$\left. \begin{array}{l} l = 2 \quad 2l + 1 = 5 \\ \quad 1 \quad 2l + 1 = 3 \\ \quad 0 \quad 2l + 1 = 1 \end{array} \right\} \begin{array}{l} \nearrow \checkmark \\ 9 \text{ dim Hilbert space} \end{array}$$

Yor did $1 + \frac{1}{2}$ in Home work.

AMS-3

To add angular momentum start with the state of largest l and largest m . It is formed by using the states of largest m_1 and m_2 . That is

$$|l = l_1 + l_2, m = l_1 + l_2\rangle = |l_1, m_1 = l_1\rangle |l_2, m_2 = l_2\rangle$$

\uparrow
 largest m
 possible

\uparrow
 largest
 m_1

\uparrow
 largest m_2

Keep applying $L_- = L_{1-} + L_{2-}$ to this state

since $[L_-, L^2] = 0$ you will not change $l = l_1 + l_2$

when you do this. However you will systematically

decrease m from $l_1 + l_2$ to $l_1 + l_2 - 1$ to $l_1 + l_2 - 2$

all the way to $-l_1 - l_2$.

Your next step is to form the state of largest

m which has $l = l_1 + l_2 - 1$. You do this by combining

$$|l_1, m_1 = l_1\rangle |l_2, m_2 = l_2 - 1\rangle \text{ and } |l_1, m_1 = l_1 - 1\rangle |l_2, m_2 = l_2\rangle$$

in a way which is orthogonal to $|l = l_1 + l_2, m = l_1 + l_2 - 1\rangle$

AMS4

From this state you can build all $l = l_1 + l_2 - 1$

states by applying L_- .

This process is continued until you exhaust

all possibilities down to $|l_1 - l_2|$

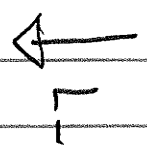
Work again through examples $l_1 = l_2 = 1$

$l_1 = l_2 = 1/2$ $l_1 = 1$ $l_2 = 1/2$ to see how it works.

A "picture" is on the next page

$$|R = R_1 + R_2 \quad m = R_1 + R_2 \rangle$$

$$= |R_1, R_1 \rangle |R_2, R_2 \rangle$$

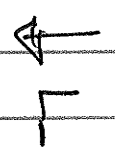


(I have not normalized the states)

$$|R = R_1 + R_2 \quad m = R_1 + R_2 - 1 \rangle$$

$$= \sqrt{2R_1} |R_1, R_1 - 1 \rangle |R_2, R_2 \rangle$$

$$+ \sqrt{2R_2} |R_1, R_1 \rangle |R_2, R_2 - 1 \rangle$$



or proportionality

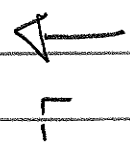
$$|R = R_1 + R_2 - 1 \quad m = R_1 + R_2 - 1 \rangle$$

$$= \sqrt{2R_2} |R_1, R_1 - 1 \rangle |R_2, R_2 \rangle$$

$$- \sqrt{2R_1} |R_1, R_1 \rangle |R_2, R_2 - 1 \rangle$$



$$|R = R_1 + R_2 \quad m = R_1 + R_2 - 2 \rangle = \dots$$



...

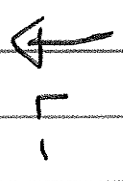
$$|R = R_1 + R_2 - 1 \quad m = R_1 + R_2 - 2 \rangle = \dots$$



...

or proportionality

$$|R = R_1 + R_2 - 2 \quad m = R_1 + R_2 - 2 \rangle = \dots$$



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