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$$[S_x, S_y] = i\hbar S_z$$

$$[S_y, S_z] = i\hbar S_x$$

$$[S_z, S_x] = i\hbar S_y$$

comm relns (but can no longer be "derived" from $L_z = x p_y - y p_x$)

SAME CALC

$$S^2 |s\ m\rangle = \hbar^2 s(s+1) |s\ m\rangle$$

$$S_z |s\ m\rangle = \hbar m |s\ m\rangle$$

$$S_\pm |s\ m\rangle = \sqrt{s(s+1) - m(m\pm 1)}\ |s\ m\pm 1\rangle$$

Spin 1/2 : two states $|\tfrac{1}{2}\ \tfrac{1}{2}\rangle \rightarrow |+\rangle$

$|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \quad |-\rangle$

This notation is much more simple but must remember missing S^2 eigenvalue label!

$$S_+ |+\rangle = 0 \qquad S_+ |-\rangle = |+\rangle$$

$$S_- |-\rangle = 0 \qquad S_- |+\rangle = |-\rangle$$

very simple!

$$\sqrt{\tfrac{1}{2}\left(\tfrac{1}{2}+1\right) - \tfrac{1}{2}\left(\tfrac{1}{2}-1\right)} = \sqrt{\tfrac{3}{4}+\tfrac{1}{4}} = 1$$

$$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle \rightarrow \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

similarly $\quad x\,\hat{x} + y\,\hat{y} \rightarrow \begin{pmatrix}x\\ y\end{pmatrix}$

Matrix for S_z

$$S_z|+\rangle = \frac{\hbar}{2}|+\rangle \qquad (m = 1/2)$$

$$S_z|-\rangle = -\frac{\hbar}{2}|-\rangle \qquad (m = -1/2)$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_+|+\rangle = 0$$
$$S_+|-\rangle = \hbar|+\rangle \qquad \text{so} \qquad S_+ = \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

<u>To be sure</u> $|+\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad S_+|+\rangle \rightarrow \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \hbar\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$|-\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad S_+|-\rangle \rightarrow \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \hbar\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\hbar|+\rangle$

✓✓

$$S_- = \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Then we can get S_x and S_y

$$S_+ = S_x + iS_y$$
$$S_- = S_x - iS_y \quad \longrightarrow \quad S_x = \frac{1}{2}(S_+ + S_-) = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$S_y = \frac{1}{2i}(S_+ - S_-) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

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PAULI SPIN MATRICES
STRIP OFF $\hbar/2$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Let's check some things

$$[S_x, S_y] \overset{\boxed{=Q}}{=} i\hbar S_z \quad ??$$

$$\left(\frac{\hbar}{2}\right)^2 \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right]$$

$$= \left(\frac{\hbar}{2}\right)^2 \left[\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \right] = \frac{\hbar^2}{2} i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i\hbar S_z \quad \checkmark$$

$$\underbrace{S_x^2 + S_y^2 + S_z^2}_{\boxed{=Q}} = \left(\frac{\hbar}{2}\right)^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

$$= \frac{3\hbar^2}{4} = \hbar \underset{\uparrow\, \ell}{\frac{1}{2}} \underset{\uparrow\, \ell+1}{\left(\frac{1}{2}+1\right)} \quad \checkmark$$

Zeeman Effect: Electron in a magnetic field

$$H = -\frac{e}{m} \vec{S} \cdot \vec{B} = -\mu_B \vec{\sigma} \cdot \vec{B}$$

$$\mu_B = \frac{e\hbar}{2m} = 5.788 \cdot 10^{-5} \frac{eV}{T}$$

What is a big magnetic field? 10T ⟵ how could you figure out?

What is a big current?

$1T = 10^4$ gauss

⟵ solenoid

B in solenoid ??

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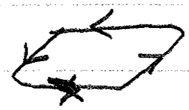
current loop in $\vec{B}$ field.



$$\text{Energy} = -\vec{\mu}\cdot\vec{B}$$

$$\mu = I A$$

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Energy levels is $\vec{B} = B\hat{z}$ in $\hat{z}$ direction?

$$H = \begin{pmatrix} -\mu_B B & 0 \\ 0 & \mu_B B \end{pmatrix}$$

Energy levels are eigenvalues of H = $-\mu_B B$
$+\mu_B B$

What if field is $B\hat{x}$ or $B\hat{y}$. Do you expect different answer? No... But looks different

$$H = \begin{pmatrix} 0 & \mu_B B \\ \mu_B B & 0 \end{pmatrix}$$

$\lambda^2 - (\mu_B B)^2 = 0$

$\lambda = \pm \mu_B B$ ✓

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$$\sigma_z \sigma_x = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\sigma_x \sigma_z = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_x \sigma_z + \sigma_z \sigma_x = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\{\sigma_x, \sigma_z\}$$

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spin 1/2 is like a fermion ??