6,

Expand $|\psi_n^1\rangle = \sum_{m\neq n} c_m^{(n)} |\psi_m^0\rangle$

$$H^0 |\psi_n^1\rangle + H^1|\psi_n^0\rangle = E_n^0 |\psi_n^1\rangle + E_n^1 |\psi_n^0\rangle$$

look at
m th component
$\langle\psi_m^0|$

$$E_m^0 c_m^{(n)} + \langle\psi_m^0|H^1|\psi_n^0\rangle$$

$$= E_n^0 c_m^{(n)} + \phi$$

$$c_m^{(n)} = \frac{\langle\psi_m^{(0)}|H^1|\psi_n^{(0)}\rangle}{E_n^0 - E_m^0}$$

(1)' ineachor Physically reasonable : Does H^1 connect ψ_n^0 to ψ_m^0?

(2) Need non-degeneracy!

$$|\psi_n^1\rangle = \sum_{m\neq n} \frac{\langle\psi_m^0|H^1|\psi_n^0\rangle}{E_n^0 - E_m^0} |\psi_m^0\rangle$$

"resonant form"

$e^{i\omega t}$

NB time dependent perturbation denominator

$$\longrightarrow \frac{1}{\omega - (E_n^0 - E_m^0)}$$

light on atom : if ω matches energy level splitting....

ASHO-2

Do anharmonic oscillator again for wf change

$$\frac{1}{4} A x^4 = \frac{1}{4}\left(\frac{\hbar}{2m\omega}\right)^2 (a+a^\dagger)^4$$

$\langle \psi_m^0 | H' | \psi_n^0 \rangle$

$m = n-4$
$n-2$
NO $m \neq n$ $\rightarrow$ n
$n+2$
$n+4$

Consider $m = n-4$

$aaaa$

$\langle \psi_{n-4}^0 | aaaa \underbrace{| \psi_n^0 \rangle}_{\text{"}|n\rangle\text{"}}$

$aaa \sqrt{n} |n-1\rangle$
$aa \sqrt{n}\sqrt{n-1} |n-2\rangle$
$a \sqrt{n}\sqrt{n-1}\sqrt{n-2} |n-3\rangle$

$\boxed{\text{DO ASHO-3 FIRST}}$

$$C_{n-4}^{(n)} = \frac{\sqrt{(n-3)(n-2)(n-1)n}}{4\hbar\omega}$$

$C_{n-2}^{(n)}$ involves what terms ?

$aaaa^\dagger$
$aaa^\dagger a$
$aa^\dagger aa$
$a^\dagger aaa$

$(n-1)\, n\, (n+1)\, (n+1)$
$(n-1)\, n \quad n\, n$
$(n-1)(n-1)\, (n-1)\, n$

ASHO-3

Do a specific n first

shift to $|\psi_1^0\rangle$ $\qquad E_1^0 = \hbar\omega(1+1/2) = 3/2\,\hbar\omega$

Connect to $|\psi_3^0\rangle$ and $|\psi_5^0\rangle$ only

$|\psi_5^0\rangle$

$a^+a^+a^+a^+|1\rangle$

$a^+a^+a^+\sqrt{2}|2\rangle$ $\qquad \dfrac{\sqrt{120}}{-2\hbar\omega} \Rightarrow -\dfrac{\sqrt{30}}{\hbar\omega}\,\dfrac{1}{4}\left(\dfrac{\hbar}{2m\omega}\right)^2$

$a^+a^+\sqrt{3}\sqrt{2}|3\rangle$

$a^+\sqrt{4}\sqrt{3}\sqrt{2}|4\rangle$

$\sqrt{5}\sqrt{4}\sqrt{3}\sqrt{2}|5\rangle$

$|\psi_3^0\rangle$

$a\,a^+a^+a^+|1\rangle$ $\qquad \dfrac{\sqrt{96}}{-\hbar\omega} \Rightarrow \dfrac{4\sqrt{6}}{-\hbar\omega}\,\dfrac{1}{4}\left(\dfrac{\hbar}{2m\omega}\right)^2$

$a\,a^+a^+\sqrt{2}|2\rangle$

$a\,a^+\sqrt{3}\sqrt{2}|3\rangle$

$a\sqrt{4}\sqrt{3}\sqrt{2}|4\rangle$

$\sqrt{4}\sqrt{4}\sqrt{3}\sqrt{2}|3\rangle$

+ 3 other terms

Degenerate Perturbation Theory:

<u>Read</u> <u>Griffiths</u>

Suppose $|\psi_a(x)\rangle$, $|\psi_b(x)\rangle$, ... all have same Energy

The first order energy shifts are eigenvalues of the matrix

$$\mathbb{V} = \left[\begin{array}{ccc} \langle\psi_a(x)|H'|\psi_a(x)\rangle & \langle\psi_a(x)|H'|\psi_b(x)\rangle & \cdots \\ \langle\psi_b(x)|H'|\psi_a(x)\rangle & \langle\psi_b(x)|H'|\psi_b(x)\rangle & \cdots \\ \vdots & \vdots & \end{array}\right.$$

W

Second order Shift:

$$E_n^2 = \sum_m \frac{\langle\psi_n^0|H'|\psi_m^0\rangle\langle\psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}$$

Note 2nd order shift of ground state is <u>always</u> negative!

$|\langle\psi_m^0|H'|\psi_n^0\rangle|^2$ but good to write out for physics

7A

Why Do 2nd order ?

(1) Most commonly because first order shift vanishes eg by symmetry

In atomic physics "selection rules"

$$\langle \psi_n^0 | \, x \, | \psi_n^0 \rangle = 0$$

odd or even in x

(2) Modern Research area (Singh)

Very high order perturbation theory

Griffiths problem 6.8 as <u>Example</u>

Consider ∞ cubical well with δ-function bump at $\left(\frac{a}{4}, \frac{a}{2}, \frac{3a}{4}\right)$

$$H' = a^3 V_0 \, \delta(x - a/4)\, \delta(y - a/2)\, \delta(z - 3a/4)$$

Find first order shift of ground state and first excited states

$$V(x,y,z) = \begin{cases} 0 & 0 < x < a \; ; \; 0 < y < a \; ; \; 0 < z < a \\ \infty & \text{otherwise} \end{cases}$$

$$\psi^0_{n_x n_y n_z}(x\, y\, z) = \left(\frac{2}{a}\right)^{3/2} \sin\frac{n_x \pi x}{a} \; \sin\frac{n_y \pi y}{a} \; \sin\frac{n_z \pi z}{a}$$

$$E^0_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$n_x, n_y, n_z = 1, 2, 3, \ldots$$

Ground state is non degenerate $\quad n_x = n_y = n_z = 1$

$$E^0_{111} = \frac{3\hbar^2\pi^2}{2ma^2}$$

$$E^1_{111} = \left(\frac{2}{a}\right)^3 a^3 V_0 \underbrace{\left(\sin\frac{\pi}{4}\right)^2}_{\frac{\sqrt{2}}{2}} \underbrace{\left(\sin\frac{\pi}{2}\right)^2}_{1} \underbrace{\left(\sin\frac{3\pi}{4}\right)^2}_{\frac{\sqrt{2}}{2}}$$

ψ_{111}^2

$$= 2V_0$$

9.

For first excited states $\quad n_x = 2 \quad n_y = n_z = 1$

$n_y = 2 \quad n_x = n_z = 1$

$n_z = 2 \quad n_x = n_y = 1$

$$
\begin{array}{cc}
 & \begin{array}{ccc} \psi_{211} & \psi_{121} & \psi_{112} \end{array} \\
\begin{array}{c} \psi_{211} \\ \\ \psi_{121} \\ \\ \psi_{112} \end{array} &
\left[\begin{array}{ccc}
\left(\sin\frac{\pi}{2}\right)^2\left(\sin\frac{\pi}{2}\right)^2\left(\sin\frac{3\pi}{4}\right)^2 & \begin{array}{c}\sin\frac{\pi}{2}\sin\frac{\pi}{2}\sin\frac{3\pi}{4} \\ \sin\frac{\pi}{4}\sin\pi\sin\frac{3\pi}{4}\end{array} & \begin{array}{c}\sin\frac{\pi}{2}\sin\frac{\pi}{2}\sin\frac{3\pi}{4} \\ \sin\frac{\pi}{4}\sin\frac{\pi}{2}\sin\frac{3\pi}{2}\end{array} \\
\sin\frac{\pi}{4}\sin\frac{\pi}{?}\sin\frac{3\pi}{4} & \left(\sin\frac{\pi}{4}\right)^2\left(\sin\pi\right)^2\left(\sin\frac{3\pi}{4}\right)^2 & \\
\begin{array}{c}\sin\frac{\pi}{4}\sin\frac{\pi}{2}\sin\frac{3\pi}{2} \\ \sin\frac{\pi}{2}\sin\frac{\pi}{2}\sin\frac{3\pi}{4}\end{array} & & \left(\sin\frac{\pi}{4}\right)^2\left(\sin\frac{\pi}{2}\right)^2\left(\sin\frac{3\pi}{2}\right)^2
\end{array}\right]
\end{array}
\left(\frac{2}{a}\right)^3 a^3 V_0
$$

$$
8V_0 \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 0 & 0 \\ -1/2 & 0 & 1/4 \end{bmatrix}
$$

Diagonalize $\quad \lambda = 0$

$$(1/2 - \lambda)(1/4 - \lambda) - 1/4 = 0$$

$$\lambda^2 - 3/4\,\lambda + 1/8 - 1/4 = 0$$

$$\lambda^2 - 3/4\,\lambda - 1/8 = 0$$

$$\lambda = \frac{3/4 \pm \sqrt{9/16 - 8/16}}{2} = \frac{3/4 \pm 1/4}{2} = 1/2, 1/4$$

E' shifts are $0, 2V_0, 4V_0$