project: M1

Lecture or ~~Homework~~?

You learned wave function $|\psi\rangle \in$ Hilbert Space

∞ dim vector space

observables $\rightarrow$ Hermitian operators $\hat{O}|\psi\rangle$

Choose basis (as always!) $|\phi_n\rangle$

$$|\psi\rangle = \sum_n c_n |\phi_n\rangle$$

If $|\phi_n\rangle$ are eigenstates of $\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle$

and $|\psi(t=0)\rangle = \sum_n c_n |\phi_n\rangle$

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_nt/\hbar} |\phi_n\rangle$$

Useful to illustrate concepts like those of perturbation theory with finite dim matrices

$$\hat{H}_0 = \begin{bmatrix} 4 & 4 & 0 \\ 4 & 10 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ in some basis

$(\lambda-4)(\lambda-10)-16=0$ $\quad E_n^0 = 2, 5, 12$

$\lambda^2 - 14\lambda + 24 = 0$

$(\lambda-2)(\lambda-12) = 0$

$\lambda = 2, 12$

project: M2

$$E_1^0 = 2 \qquad |\phi_1^0\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$E_2^0 = 5 \qquad |\phi_2^0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$E_3^0 = 12 \qquad |\phi_3^0\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} \qquad \alpha, \beta, \gamma \ll 1$$

Perturbation theory

$$\Delta E_1 = \langle \phi_1 | \hat{V} | \phi_1 \rangle = \frac{1}{5}\,(2 \;\; -1 \;\; 0)\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{5}\,(2 \;\; -1 \;\; 0)\begin{pmatrix} 2\alpha \\ -\beta \\ 0 \end{pmatrix} = \frac{1}{5}(4\alpha + \beta)$$

$$\Delta E_2 = \langle \phi_2 | \hat{V} | \phi_2 \rangle = (0 \;\; 0 \;\; 1)\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \gamma$$

$$\Delta E_3 = \ldots$$

project
M3

Check by direct computation

$$\hat{H} = \hat{H}_0 + \hat{V} = \begin{pmatrix} 4+\alpha & 4 & 0 \\ 4 & 10+\beta & 0 \\ 0 & 0 & 5+\gamma \end{pmatrix}$$

clearly one eigenvalue is $5+\gamma$

(first order perturbation theory is exact!)

$$(4+\alpha-\lambda)(10+\beta-\lambda) - 16 = 0$$

$$\lambda^2 - \lambda(14+\alpha+\beta) + 24 + 10\alpha + 4\beta + \alpha\beta = 0$$

exact new eigenvalues:

$$E = \frac{1}{2}\left[(14+\alpha+\beta) \pm \sqrt{(14+\alpha+\beta)^2 - 4(24+10\alpha+4\beta+\alpha\beta)}\right]$$

$$= \frac{1}{2}\left[(14+\alpha+\beta) \pm \sqrt{196+28\alpha+28\beta+2\alpha\beta+\alpha^2+\beta^2 - 96 - 40\alpha - 16\beta - 4\alpha\beta}\right]$$

$$100 - 12\alpha + 12\beta - 2\alpha\beta + \alpha^2 + \beta^2$$

DiVOGA

project: M4 Exact

$$E = \frac{1}{2}\left[(14+\alpha+\beta) \pm \sqrt{100-12\alpha+12\beta-2\alpha\beta+\alpha^2+\beta^2}\right]$$

Now Expand

$$\approx \left[100-12\alpha+12\beta\right]^{1/2}$$

$$= 10\left[1-\frac{3}{25}\alpha+\frac{3}{25}\beta\right]^{1/2}$$

$$\approx 10\left[1-\frac{3}{50}\alpha+\frac{3}{50}\beta\right]$$

$$E \approx \frac{1}{2}\left[14+\alpha+\beta \pm \left(10-\frac{3}{5}\alpha+\frac{3}{5}\beta\right)\right]$$

$$E_+ \approx \frac{1}{2}\left[14+\alpha+\beta+10-\frac{3}{5}\alpha+\frac{3}{5}\beta\right]$$

$$= 12+\frac{1}{5}\alpha+\frac{4}{5}\beta$$

shift is $\frac{1}{5}\alpha+\frac{4}{5}\beta$ in $E_3 = 12$

$$E_- \approx \frac{1}{2}\left[14+\alpha+\beta-10+\frac{3}{5}\alpha-\frac{3}{5}\beta\right]$$

$$= 2+\frac{4}{5}\alpha+\frac{1}{5}\beta$$

shift in $E_1 = 2$ is $\frac{4}{5}\alpha+\frac{1}{5}\beta$ ✓✓