AM-19

Two spin 1/2 objects (A start on learning how to add angular momentum)

Why not an issue before $\vec{p}_1 + \vec{p}_2$ is difficult?

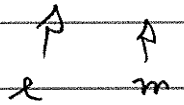
No! because p_x, p_y, p_z commute!

Trickier to do $\vec{L}_1 + \vec{L}_2$ because $L_x\ L_y\ L_z$ do not commute!

Basis

$|1/2\ 1/2\rangle \rightarrow |+\rangle$

$|1/2\ -1/2\rangle \rightarrow |-\rangle$



Two spin 1/2 objects

$|l_1\ l_2\ m_1\ m_2\rangle$

$|1/2\ 1/2\ 1/2\ 1/2\rangle \rightarrow |++\rangle$

$|+-\rangle$

$|-+\rangle$

$|--\rangle$

eg $S_1^2 |+-+\rangle = 3/4\, \hbar^2 |++\rangle$

$S_1^z |+-\rangle = 1/2\, \hbar |+-\rangle$

$S_2^z |+-\rangle = -1/2\, \hbar |+-\rangle$

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$$(S_1^z + S_2^z)\,|++\rangle = 1\hbar\,|++\rangle$$

$$|+-\rangle = 0\hbar\,|+-\rangle$$

$$|-+\rangle = 0\hbar\,|-+\rangle$$

$$|--\rangle = -\hbar\,|--\rangle$$

okay... so what?

Better clue as to what is going on

$$(S_1+S_2)^2 = S_1^2 + 2\vec{S}_1\cdot\vec{S}_2 + S_2^2$$

$$S_1^+ = S_1^x + iS_1^y$$

$$S_1^- = S_1^x - iS_1^y$$

What will we do with xy??

$$S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z$$

$$\frac{1}{4}\left(S_1^+ + S_1^-\right)\left(S_2^+ + S_2^-\right) - \frac{1}{4}\left(S_1^+ - S_1^-\right)\left(S_2^+ - S_2^-\right)$$

$$= \frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right)$$

$$(S_1+S_2)^2\,|++\rangle = \frac{3}{4}\hbar^2\,|++\rangle + 0 + 2\frac{1}{4}\hbar^2\,|+-\rangle + \frac{3}{4}\hbar^2\,|++\rangle$$

S_1^2 | $+/-$ | zz | S_2^2

$2\vec{S}_1\cdot\vec{S}_2$

$$= 2\hbar^2\,|++\rangle = s(s+1)\hbar^2 \quad \text{for } s=1$$

Similarly $(S_1+S_2)^2\,|--\rangle = 2\hbar^2\,|--\rangle$

But

+/- term acts now!

BA $(S_1+S_2)^2 |+-\rangle = \hbar^2 |+-\rangle + \hbar^2 |-+\rangle$

$(S_1+S_2)^2 |-+\rangle = \hbar^2 |-+\rangle + \hbar^2 |+-\rangle$

$(S_1+S_2)^2 \left[\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)\right]$

$= 2\hbar^2 \left[\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)\right]$

$(S_1+S_2)^2 \left[\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)\right]$

$= 0 \left[\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)\right]$

Now look at this table which summarizes

	$(S_1+S_2)^2$	"simple"	$S_1^z + S_2^z$	
$\|++\rangle$	$1(1+1)\hbar^2$		$1\hbar$	angular momentum $\ell = 1$ $m = (-1, 0, 1)$
$\frac{1}{\sqrt{2}}(\|+-\rangle + \|-+\rangle)$	$1(1+1)\hbar^2$	"symmetric"	$0\hbar$	
$\|--\rangle$	$1(1+1)\hbar^2$		$-1\hbar$	
$\frac{1}{\sqrt{2}}(\|+-\rangle - \|-+\rangle)$	$0(0+1)\hbar^2$	"singlet"	$0\hbar$	angular momentum $\ell = 0$ $m = 0$

$1/2 + 1/2 = 1 + 0$