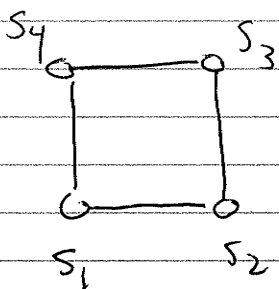


45H-1

## Four Site Heisenberg Model

$$H = \frac{J}{\hbar^2} (S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_4 + S_4 \cdot S_1)$$



$$= \frac{J}{2\hbar^2} \left[ (S_1 + S_2 + S_3 + S_4)^2 - (S_1 + S_3)^2 - (S_2 + S_4)^2 \right]$$

$$S_1 + S_3 = \begin{cases} 0 \\ 1 \end{cases} \quad S_2 + S_4 = \begin{cases} 0 \\ 1 \end{cases}$$

$S_1 + S_3$	$S_2 + S_4$	$S_1 + S_2 + S_3 + S_4$	# states
0	0	0	1
0	1	1	3
1	0	1	3
1	1	0	1
		1	3
		2	5
			16

checks =  $2^4$  ! }

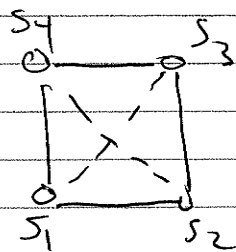
454-2

$S_1 + S_3$	$S_2 + S_4$	$S_1 + S_2 + S_3 + S_4$	E
0	0	0	$\frac{J}{2\hbar^2} [0 - 0 - 0]$
0	1	1	$\frac{J}{2\hbar^2} [2 - 0 - 2] \hbar^2$
1	0	1	$\frac{J}{2\hbar^2} [2 - 2 - 0] \hbar^2$
1	1	0	$\frac{J}{2\hbar^2} [0 - 2 - 2] \hbar^2$
		1	$\frac{J}{2\hbar^2} [2 - 2 - 2] \hbar^2$
		2	$\frac{J}{2\hbar^2} [6 - 2 - 2] \hbar^2$

i. Energy levels are

$-2J$	1	← ground state
$-J$	3	is total spin
0	7	
$+J$	5	

Generalize to



"Frustrated"

$$\frac{J}{\hbar^2} [S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_4 + S_4 \cdot S_1] + \frac{J'}{\hbar^2} (S_1 \cdot S_3 + S_2 \cdot S_4)$$

Try to form from  $(S_1 + S_2 + S_3 + S_4)^2$       A

$(S_1 + S_3)^2 + (S_2 + S_4)^2$       B

$S_1^2 + S_2^2 + S_3^2 + S_4^2$       C

454-3

Equivalently

$$\frac{J}{h^2} = 2A$$

$$A = \frac{J}{2h^2}$$

$$\frac{J'}{h^2} = 2A + 2B$$

$$B = \frac{J' - J}{2h^2}$$

$$0 = A + B + C$$

$$C = -\frac{J'}{2h^2}$$

That is

$$\mathcal{H} = \frac{J}{2h^2} (S_1 + S_2 + S_3 + S_4)^2 + \frac{J' - J}{2h^2} [(S_1 + S_3)^2 + (S_2 + S_4)^2]$$

$$= \frac{J'}{2h^2} [S_1^2 + S_2^2 + S_3^2 + S_4^2]$$

check limits (1)  $J' = 0$  Agrees with 454-1

(2)  $J = 0$  Agrees with Isik Hagenberg (2 copies)

$S_1 + S_3$	$S_2 + S_4$	$S_1 + S_2 + S_3 + S_4$	E
0	0	0	$-\frac{J'}{2h^2} 4 \cdot 3/4 = -3J'/2$
0	1	1	$+J + (J' - J) - 3J'/2 = -J'/2$
1	0	1	$= -J'/2$
1	1	0	$(J' - J)2 - 3J'/2 = J'/2 - 2J$
		1	$J + 2(J' - J) - 3J'/2 = J'/2 - J$
		2	$3J + 2(J' - J) - 3J'/2 = J'/2 + J$

45/11 ~~11~~

For  $J' = 0$

$$E_0 = -2J$$

as we saw earlier

As  $J'$  increases:

