Exchange

There is an energy associated with the antisymmetry requirement, as we have seen $KE_{fermion} \gg KE_{distinguishable}$

There is another way to think about this

$$\psi(x_1, x_2) = \psi_a(x_1)\psi_b(x_2) \qquad \text{Distinguishable} \quad (1)$$

$$= \frac{1}{\sqrt{2}}\left[\psi_a(x_1)\psi_b(x_2) + \psi_a(x_2)\psi_b(x_1)\right] \qquad \text{Bosons} \quad (2)$$

$$= \left[\qquad - \qquad\right] \qquad \text{Fermion} \quad (3)$$

(1) $\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle$

(1) $\langle x_1^2 \rangle = \int x_1^2 |\psi_a(x_1)|^2 dx_1 \int |\psi_b(x_2)|^2 dx_2 = \langle x^2 \rangle_a$

$\langle x_2^2 \rangle = \langle x^2 \rangle_b$

$\langle x_1 x_2 \rangle = \langle x \rangle_a \langle x \rangle_b$

$\langle (x_1 - x_2)^2 \rangle = \langle x^2 \rangle_a + \langle x^2 \rangle_b - 2\langle x \rangle_a \langle x \rangle_b$

(2)
(3)

$$\langle x_1^2 \rangle = \frac{1}{2}\Big[\int x_1^2 |\psi_a(x_1)|^2 dx_1 \int |\psi_b(x_2)|^2 dx_2$$

$$+ \int x_1^2 |\psi_b(x_1)|^2 dx_1 \int |\psi_a(x_2)|^2 dx_2$$

$$\pm \int x_1^2 \psi_a^*(x_1)\psi_b(x_1)\,dx_1 \int \psi_b^*(x_2)\psi_a(x_2)\,dx_2$$

$$\pm \int x_1^2 \psi_b^*(x_1)\psi_a(x_1)\,dx_1 \int \psi_a^*(x_2)\psi_b(x_2)\,dx_2\Big]$$

$$= \frac{1}{2}\left[\langle x^2\rangle_a + \langle x^2\rangle_b \pm 0 \pm 0\right] \leftarrow \text{orthogonal}$$

Same for $\langle x_2^2 \rangle$ of course, since indistinguishable

$$\langle x_1 x_2 \rangle = \frac{1}{2}\Big[\int x_1 |\psi_a(x_1)|^2 dx_1 \int x_2 |\psi_b(x_2)|^2 dx_2$$

$$+ \ldots$$

$$= \langle x\rangle_a \langle x\rangle_b \pm |\langle x\rangle_{ab}|^2$$

$$\int x\, \psi_a^*(x)\,\psi_b(x)\,dx$$

$$\langle (x_1 - x_2)^2 \rangle = \underbrace{\langle x\rangle_a^2 + \langle x\rangle_b^2}_{\text{Distinguishable answer}} - 2\langle x\rangle_a \langle x\rangle_b \mp 2|\langle x\rangle_{ab}|^2$$

quantum effect

Separation lower for bosons, higher for fermions

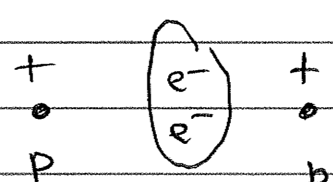
$\langle x \rangle_{ab} = 0$ unless wave functions overlap

(Do not need to worry about e^- on earth affecting e^- on the moon!)

"Exchange force" bosons clump
fermions repel

purely QM purely from symmetry/antisymmetry

Hydrogen molecule (covalent bond)



symmetric (spatial) — molecule forms

antisymmetric (spatial) — molecule unstable

So why is H stable is fermions → antisymmetric

It is because e^- are in spin singlet and have symmetric spatial wf.

singlet → bonding ← covalent bond requires spin singlet

triplet → antibonding