SOLUTIONS

PROBLEM SET 1 Due Friday September 30

Physics 115B– FALL 2011

Points assigned

Analytic:

10 [**1.**] Griffiths Problem 6.1

10 [**2.**] Griffiths Problem 6.2

10 [**3.**] Griffiths Problem 6.5

20 [**4.**] Griffiths Problem 6.7

20 [**5.**] Griffiths Problem 6.9

10 [**6.**] In deriving perturbation theory we used the fact that the left and right eigenvectors and eigenvalues of H^0 are identical:

$$\langle \Psi_n^0 | H^0 = \langle \Psi_n^0 | E_n^0 \quad \Leftrightarrow \quad H^0 | \Psi_n^0 \rangle = E_n^0 | \Psi_n^0 \rangle$$

What property of H^0 makes this true? Consider the matrix

$$M = \begin{pmatrix} 0.5 & 0.7 \\ 0.5 & 0.3 \end{pmatrix}$$

Show that the vector $\vec{V} = \begin{pmatrix} 1 & 1 \end{pmatrix}$ is a left eigenvector but not a right eigenvector. Why does M fail to have identical left and right eigenvectors? Are the left and right eigenvalues of M the same? Why or why not?

Numeric:

Comment: For the first part of the course, as you develop skill in programming, the computational problems will not necessarily have anything to do with quantum mechanics.

10 [**7.**] Write a C program to sum a geometric series

$$S = a + ar + ar^2 + ar^3 + \cdots ar^N$$

Hand the program in with some output which demonstrates that it is working properly.

10 [**8.**] Write a C program to compute the Fibonacci numbers, which are generated by the recursion relation

$$f_{n+2} = f_n + f_{n+1}$$

with $f_1 = f_2 = 1$. Hand the program in with some output which demonstrates that it is working properly.

Physics 115B - HW#1 SOLUTIONS

#1 Griffiths Prob. 6.1

Perturbation = $H' = \alpha\,\delta(x - a/2)$ to infinite sq. well $\quad \psi_n^0(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi n}{a}x\right)$

$$E_n^0 = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2\pi^2 n^2}{2ma^2} \quad \text{where } k = \frac{n\pi}{a}$$

(a) $$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle = \frac{2\alpha}{a}\int_0^a \sin^2\left(\frac{\pi n}{a}x\right)\delta\left(x - \frac{a}{2}\right)dx = \boxed{\frac{2\alpha}{a}\sin^2\left(\frac{\pi n}{2}\right)}$$

For even n: $E_n^1 = \frac{2\alpha}{a}\sin^2(\pi) = 0$, $\psi_n(x)$ goes through point $(a/2, 0)$ and is not perturbed.

(b) $$|\psi_1^1\rangle = \sum_{m\neq n} \frac{\langle m | H' | 1\rangle}{E_1^0 - E_m^0}|\psi_m^0\rangle \quad \text{where } |m\rangle = |\psi_n^0\rangle$$

Now: $$E_1^0 - E_m^0 = \frac{\pi^2\hbar^2}{2ma^2}(1 - m^2)$$

$$\langle \psi_1^0 | H' | \psi_m^0\rangle = \frac{2}{a}\int_0^a \sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{2m\pi}{a}x\right)\alpha\,\delta\left(x - \frac{a}{2}\right)dx$$

$$= \frac{2}{a}\sin\left(\frac{\pi}{2}\right)\sin(m\pi)$$

All even terms $(m = 2, 4, 6..) \Longrightarrow \sin(m\pi) = 0$

so, include only 1st (3) odd terms (3, 5, 7)

$$|\psi_1^1\rangle = \frac{\langle \psi_1^0 | H' | \psi_3^0\rangle}{E_1^0 - E_3^0}|\psi_3^0\rangle + \frac{\langle \psi_1^0 | H' | \psi_3^0\rangle}{E_1^0 - E_5^0}|\psi_5^0\rangle + \frac{\langle \psi_1^0 | H' | \psi_5^0\rangle}{E_1^0 - E_5^0}|\psi_5^0\rangle$$

$$= \left(\frac{2\alpha}{a}\right)\left(\frac{2ma^2}{\pi^2\hbar^2}\right)\sqrt{\frac{2}{a}}\left[\frac{1}{8}\sin\left(\frac{3\pi}{a}x\right) - \frac{1}{24}\sin\left(\frac{5\pi x}{a}\right) + \frac{1}{48}\sin\left(\frac{7\pi}{a}x\right)\right]$$

$$\boxed{|\psi_1^1\rangle = \frac{m\alpha}{\pi^2\hbar^2}\sqrt{\frac{a}{2}}\left[\sin\left(\frac{3\pi}{a}x\right) - \frac{1}{3}\sin\left(\frac{5\pi}{a}x\right) + \frac{1}{6}\sin\left(\frac{7\pi}{a}x\right)\right]}$$

#2 Griffiths Prob 6.2

For $E_n = (n+\frac{1}{2})\hbar\omega$ (HO) Let $k \to (1+\varepsilon)k$

where $\omega = \sqrt{\frac{k}{m}}$

a) $\omega \longrightarrow \omega(1+\varepsilon)^{1/2}$

Power series exp.

$$E_n' = (n+\tfrac{1}{2})\hbar\omega(1+\varepsilon)^{1/2} = \boxed{(n+\tfrac{1}{2})\hbar\omega\left[1+\boxed{\varepsilon/2}-\tfrac{1}{8}\varepsilon^2+\dots\right]}$$

b) $H_0 = -\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{X}^2$ $H'(\text{pert}) = \frac{1}{2}m\omega^2\varepsilon\hat{X}^2$

Use const/destruct operators a^+ and a:

$$\hat{X} = \sqrt{\frac{\hbar}{2m\omega}}(a^+ + a) \qquad \hat{X}^2 = \frac{\hbar}{2m\omega}\left((a^+)^2 + a^+a + aa^+ + a^2\right)$$

$$E_n^1 = \langle n|\tfrac{1}{2}m\omega^2\varepsilon\hat{X}^2|n\rangle = \left(\frac{m\omega^2}{2}\varepsilon\right)\left(\frac{\hbar}{2m\omega}\right)\langle n|a^+a + aa^+|n\rangle$$

$$= \frac{\hbar\omega}{4}\varepsilon\left[\langle n|a^+a|n\rangle + \langle n|aa^+|n\rangle\right] \qquad \begin{cases} a^+|n\rangle = \sqrt{n+1}\,|n\rangle \\ a\,|n\rangle = \sqrt{n}\,|n-1\rangle \end{cases}$$

$$= \frac{\hbar\omega}{4}\varepsilon(n+n+1) = \boxed{\frac{\varepsilon}{2}\hbar\omega(n+1/2)}$$

Same as 1st order term in expansion in a.

#3 Griffiths Problem 6.5

Charged particle in HM 1D Pot'l : $H' = -q\mathcal{E}x$

(a) $E_n^1 = \langle n|H'|n\rangle$ Again use $\hat{X} = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a)$

$$= -q\mathcal{E}\sqrt{\frac{\hbar}{2m\omega}}\left[\langle n|a^\dagger|n\rangle^{\nearrow 0} + \langle n|a|n\rangle^{\nearrow 0}\right] = 0$$

$$E_n^2 = \sum_{m\neq n} \frac{\langle m|H'|n\rangle^2}{E_n^0 - E_m^0}$$

From 3.3 (Hint), we have: $\langle n|X|n'\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n'}\,\delta_{n n'-1} + \sqrt{n}\,\delta_{n' n-1}\right)$

$$\langle m|H'|n\rangle = -q\mathcal{E}\langle m|\hat{X}|n\rangle = -q\mathcal{E}\sqrt{\frac{\hbar}{2m\omega}}\begin{cases}\sqrt{n+1} & \text{for } m = n+1\\ \sqrt{n-1} & \text{for } m = n-1\\ 0 & \text{otherwise}\end{cases}$$

$$E_n^0 - E_{n+1}^0 = -\hbar\omega, \quad E_n^0 - E_{n-1}^0 = \hbar\omega$$

$$\Rightarrow E_n^2 = \frac{q^2\mathcal{E}^2\hbar}{2m\omega}\left[\frac{n+1}{-\hbar\omega} + \frac{n}{\hbar\omega}\right] = \frac{q^2\mathcal{E}^2}{2m\omega}(-1) = \boxed{\frac{-q^2\mathcal{E}^2}{2m\omega^2}}$$

(b) SE: $\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2 - q\mathcal{E}x\right]\psi = E\psi$

Let $X' = X - \frac{q\mathcal{E}}{m\omega^2}$

So: $\frac{1}{2}m\omega^2 X^2 - q\mathcal{E}X = \frac{1}{2}m\omega^2\left(X' + \frac{q\mathcal{E}}{m\omega^2}\right)^2 - q\mathcal{E}\left(X' + \frac{q\mathcal{E}}{m\omega^2}\right)$

$$= \frac{1}{2}m\omega^2\left(X'^2 + \frac{2q\mathcal{E}}{m\omega^2}X' + \frac{q^2\mathcal{E}^2}{m^2\omega^4}\right) - q\mathcal{E}X' - \frac{q^2\mathcal{E}^2}{m\omega^2}$$

terms cancel

$$= \frac{1}{2}m^2 X'^2 - \frac{1}{2}\frac{q^2\mathcal{E}^2}{m\omega^2}$$

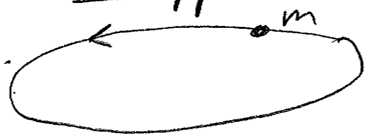
New SE $\left[\frac{-\hbar^2}{2m}\frac{d}{dx^2} + \frac{1}{2}m^2 X'^2\right]\psi = \left[E + \frac{1}{2}\frac{q^2\mathcal{E}^2}{m\omega^2}\right]\psi$

Soln for SHM is: $E_n + \frac{1}{2}\frac{q^2\mathcal{E}^2}{m\omega^2} = \hbar\omega(n + \frac{1}{2})$

or $E_n = \hbar\omega(n + 1/2) - \boxed{\frac{1}{2}\frac{q^2\mathcal{E}^2}{m\omega^2}}$ Same correction as part (a)

#4 Griffiths 6.7

(a)



SE: $\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi$

Soln: $\psi(x) = Ae^{ikx}$ $\quad k = \pm\sqrt{\frac{2mE}{\hbar^2}}$

But $\psi(x+L/2) = \psi(x-L/2) \Rightarrow$

$$Ae^{ikL/2} = Ae^{-ikL/2} \Rightarrow e^{ikL} = 1 \Rightarrow \boxed{kL = 2\pi n}$$

So $\psi(x) = A\, e^{\frac{i2\pi n}{L}x}$ $\quad$ Normalize $\Rightarrow A = 1/\sqrt{L}$

$$\boxed{\psi(x) = \frac{1}{\sqrt{L}}e^{\frac{i2\pi n x}{L}} \quad \left(-\frac{L}{2} < x < \frac{L}{2}\right) \quad n = 0, \pm 1, \pm 2.}$$

$$\boxed{E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{2\pi n}{L}\right)^2 = \frac{2}{m}\left(\frac{n\pi\hbar}{L}\right)^2 = E_{-n}}$$

(b) Perturbation: $H' = -V_0 e^{-x^2/a^2}$ $\quad a \ll L$

$$W_{aa} = \langle\psi_n|H'|\psi_n\rangle = \frac{1}{L}\int_{-L/2}^{L/2} -V_0 e^{-x^2/a^2}dx = \frac{-V_0}{L}\int_{-\infty}^{\infty} e^{-x^2/a^2}dx$$

$$= -\frac{V_0}{L}\sqrt{\pi/(1/a^2)} = \boxed{\frac{-V_0 a}{L}\sqrt{\pi}} = W_{bb} = \langle\psi_{-n}|H'|\psi_{-n}\rangle$$

$$W_{ab} = \langle\psi|H'|\psi_{-n}\rangle = \frac{-V_0}{L}\int_{-\infty}^{\infty} e^{-x^2/a^2} e^{-\frac{i4\pi n x}{L}}dx$$

$$= \frac{-V_0}{L}\left[\int_{-\infty}^{\infty} e^{-\frac{\left(x - \frac{i2\pi n a}{L}\right)^2}{a^2} - \left(\frac{2\pi n a}{L}\right)^2}\right] = -\frac{V}{L}e^{-\left(\frac{2\pi n a}{L}\right)^2}\int_{-\infty}^{\infty} e^{-x'^2/a^2}dx' \quad \left(x' = x - \frac{i2\pi n a}{L}\right)$$

$$\boxed{W_{ab} = -\frac{V_0}{L}e^{-\left(\frac{2\pi n a}{L}\right)^2} a\sqrt{\pi}}$$

4 Griffiths 6.7

$$E^1_\pm = \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right]$$

(with $W_{aa} - W_{bb} = 0$)

$$= \frac{1}{2}\left[2\left(\frac{-V_0 a\sqrt{\pi}}{L}\right) \pm 2\left(-\frac{V_0}{L} e^{-\left(\frac{2\pi n a}{L}\right)^2} a\sqrt{\pi}\right)\right]$$

$$\boxed{\begin{aligned} E^1_+ &= -\frac{V_0 a\sqrt{\pi}}{L}\left(1 - e^{-\left(\frac{2\pi n a}{L}\right)^2}\right) \\ E^1_- &= -\frac{V_0 a\sqrt{\pi}}{L}\left(1 + e^{-\left(\frac{2\pi n a}{L}\right)^2}\right) \end{aligned}}$$

(c) Find "good" linear comb. of $|\psi_n\rangle$ and $|\psi_{-n}\rangle$

From: $\alpha W_{aa} + \beta W_{bb} = \alpha E^1 \Rightarrow \beta = \alpha\left[\frac{E^1 - W_{aa}}{W_{ab}}\right]$

$$\text{So } \beta = \left[\frac{\pm \frac{V_0 a\sqrt{\pi}}{L} e^{-\left(\frac{2\pi n a}{L}\right)^2}}{-\frac{V_0 a\sqrt{\pi}}{L} e^{-\left(\frac{2\pi n a}{L}\right)^2}}\right]\alpha = \boxed{\pm\alpha}$$

$$|\psi_1^{good}\rangle = \alpha|\psi_n\rangle - \alpha|\psi_{-n}\rangle = \frac{\alpha}{\sqrt{L}}\left(e^{i\frac{2\pi n a}{L}} - e^{-i\frac{2\pi n a}{L}}\right)$$

$$= \frac{\alpha}{\sqrt{L}} 2i\sin\left(\frac{2\pi n a}{L}\right) = \boxed{\sqrt{\frac{2}{L}}\, i\sin\left(\frac{2\pi n a}{L}\right)} \qquad \alpha = \frac{1}{\sqrt{2}}$$

$$|\psi_2^{good}\rangle = \alpha|\psi_n\rangle + \alpha|\psi_n\rangle = \frac{\alpha}{\sqrt{L}}\left(e^{i\frac{2\pi n a}{L}} + e^{-i\frac{2\pi n a}{L}}\right)$$

$$= \boxed{\sqrt{\frac{2}{L}}\cos\left(\frac{2\pi n a}{L}\right)}$$

Note: $\boxed{\begin{aligned} i\sin x &= \frac{e^{ix} - e^{-ix}}{2} \\ \cos x &= \frac{e^{ix} + e^{-ix}}{2} \end{aligned}}$

(continued)

(#4 Griffiths 6.7 (c) continued

$$E_+^1 = \langle \psi_+ | H' | \psi_+ \rangle = \left(\frac{2}{L}\right)\int_{-L/2}^{L/2} dx \left(-i\sin\left(\frac{2\pi n x}{L}\right)\right)\left(-V_0 e^{-x^2/a^2}\right)\left(i\sin\left(\frac{2\pi n x}{L}\right)\right)$$

$$= -\frac{2V_0}{L}\int_{-L/2}^{L/2} e^{-x^2/a^2}\sin^2\left(\frac{2\pi n x}{L}\right) dx$$

$$E_-^1 = \langle \psi_- | H' | \psi_- \rangle = \left(\frac{2}{L}\right)\int_{-L/2}^{L/2} dx \cos\left(\frac{2\pi n x}{L}\right)\left(-V_0 e^{-x^2/a^2}\right)\cos\left(\frac{2\pi n x}{L}\right)$$

$$= -\frac{2V_0}{L}\int_{-L/2}^{L/2} e^{-x^2/a^2}\cos^2\left(\frac{2\pi n x}{L}\right) dx$$

Extend limits of integration $(L/2, -L/2)$ to $(\infty, -\infty)$ since $a \ll L$ and $e^{-L^2/a^2} \approx 0$.

Also, use double angle formulas:

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1}{2}(1 - \cos 2x)$
and $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$$E_+^1 = -\frac{2V_0}{L}\int_{-\infty}^{\infty} e^{-x^2/a^2}\frac{1}{2}\left(1 - \cos\left(\frac{4\pi n x}{L}\right)\right) dx$$

$$= -\frac{V_0}{L}\left[\underbrace{\int_{-\infty}^{\infty} e^{-x^2/a^2}}_{(1)} - \underbrace{\int_{-\infty}^{\infty} e^{-x^2/a^2}\cos\left(\frac{4\pi n x}{L}\right) dx}_{(2)}\right]$$

Note: $\cos x = \frac{e^{ix} + e^{-ix}}{2} \approx e^{-ix}$ for $[-\infty, \infty]$

So (1) $= a\sqrt{\pi}$ (from b) and (2) $= \int_{-\infty}^{\infty} e^{-x^2/a^2} e^{-i4\pi n x/L} dx = a\sqrt{\pi}\, e^{-\left(\frac{2\pi n a}{L}\right)^2}$ (from b also)

$$E_+ = -\frac{V_0}{L} a\sqrt{\pi}\left[1 - e^{-\left(\frac{2\pi n a}{L}\right)^2}\right]$$

Same as part (b)

Using same logic

$$E_-^1 = -\frac{V_0}{L} a\sqrt{\pi}\left[1 + e^{-\left(\frac{2\pi n a}{L}\right)^2}\right]$$

(9)

#4 Griffiths 6.7 continued

(d) Find Hermitian operator A where $[A, H_0] = [A, H'] = 0$

The parity operator: $P\psi(x) = \psi(-x)$

would work.

$[P, H_0] = 0$ and $[P, H'] = 0$ } Check it out!

The eigenvalues of P are ± 1 which correspond to even (cos) and odd (sin) eigenstates (functions) of H_0. So "good" eigenstates are just the even/odd functions of H_0:

$$|\psi_n^{odd}\rangle = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi n x}{L}\right)$$

$$|\psi_n^{even}\rangle = \sqrt{\frac{2}{L}} \cos\left(\frac{2\pi n x}{L}\right)$$

} same as (c)

(10)

#5 Griffiths 6.9

$$H = V_0\begin{pmatrix} 1-\varepsilon & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix} = V_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} + V_0\begin{pmatrix} -\varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon \\ 0 & \varepsilon & 0 \end{pmatrix}$$

$$(H_0) \quad + \quad (H')$$

(a) Eigenvalues/vectors of H_0 (unperturbed) can be read directly: Eigenvalues are: V_0, V_0 (degenerate), $2V_0$

Eigenvectors: $|1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $|2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $|3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(b) Exact eigenvalues of H:

$$\det(H - \lambda I) = (1-\varepsilon-\lambda)[(1-\lambda)(2-\lambda)-\varepsilon^2] = (1-\varepsilon-\lambda)[(2-\varepsilon^2)-3\lambda+\lambda^2]$$

$$\boxed{\lambda_1 = (1-\varepsilon)V_0} \qquad \lambda_{2,3} = \frac{3 \pm \sqrt{9-8+4\varepsilon^2}}{2} = \frac{3}{2} \pm \frac{1}{2}\sqrt{1+4\varepsilon^2}$$

$$\lambda_{2,3} = V_0\left[\frac{3}{2} \pm \frac{1}{2}(1+2\varepsilon^2)\right] \Rightarrow \boxed{\begin{matrix} \lambda_2 = V_0(1-\varepsilon^2) \\ \lambda_3 = V_0(2+\varepsilon^2) \end{matrix}}$$

(c) Non-degenerate eigenvector of $H^0 = |3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$E_3^1 = \langle 3|H'|3\rangle = (0\ 0\ 1)\, V_0\begin{pmatrix} -\varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon \\ 0 & \varepsilon & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \boxed{0} \quad \text{NO 1st order correction}$$

$$E_3^2 = \sum_{m \neq n} \frac{\langle m|H'|n\rangle^2}{E_n^0 - E_m^0} = \frac{\langle 1|H'|3\rangle^2}{2V_0 - V_0} + \frac{\langle 2|H'|3\rangle^2}{2V_0 - V_0}$$

$$\langle 1|H'|3\rangle = (1\ 0\ 0)\, V_0\begin{pmatrix} -\varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon \\ 0 & \varepsilon & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -\varepsilon V_0$$

$$\langle 2|H'|3\rangle = (0 1 0) \quad \text{"} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$E_3^2 = \frac{\varepsilon^2 V_0^2}{V_0} + \frac{0}{V_0} = \boxed{\varepsilon^2 V_0}$$

$$E_3 = E_3^0 + E_3^1 + E_3^2 = 2V_0 + 0 + \varepsilon^2 V_0 = \boxed{V_0(2+\varepsilon^2)}$$

same as (b)

5 Griffiths 6.9 (d)

(d) $W_{aa} = \langle 1|H'|1\rangle = (1\ 0\ 0)\,\varepsilon V_0 \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -\varepsilon V_0$

$W_{bb} = \langle 2|H'|2\rangle = (0\ 1\ 0) \quad '' \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$

$W_{ab} = W_{ba} = \langle 1|H'|2\rangle = (1\ 0\ 0) \quad '' \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$

$E^1_{\pm} = \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 - 4|W_{ab}|^2}\right]$

$= -\frac{\varepsilon}{2} V_0 \pm \frac{\varepsilon}{2} V_0 = \begin{Bmatrix} -\varepsilon V_0 \\ 0 \end{Bmatrix}$

$E_1 = E_1^0 + E_1^1 = V_0 - \varepsilon V_0 = \boxed{V_0(1-\varepsilon)}$ Correct to

$E_2 = E_2^0 + E_2^1 = V_0 - 0 = \boxed{V_0}$ 1st order

#6 H^0 hermetian property ensures this fact

$$H_0|\Psi_n\rangle = E_0|\Psi_n\rangle \Rightarrow (H_0|\Psi_n\rangle)^* = (E_n^0|\Psi_n\rangle)^*$$

$$\Rightarrow \langle\Psi_n|H_0^+ = \langle\Psi_n|H_0 = \langle\Psi_n|E_n^0 \quad (E_n^0 = E_n^{0*}) \text{ (real)}$$

definition of hermitean: $H_0^+ = H_0$

Let $M = \begin{pmatrix} .5 & .7 \\ .5 & .3 \end{pmatrix}$ $V = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$V^T M = (1\ 1)\begin{pmatrix} .5 & .7 \\ .5 & .3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ So $\bar{V}$ is left eigenvector of M

$$M\ V = \begin{pmatrix} .5 & .7 \\ .5 & .3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.2 \\ .8 \end{pmatrix} \neq \alpha\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ $\bar{V}$ is not right eigenvector of M

Reason: M is not symmetric, that is $M \neq M^T$

If M were symmetric and V is left eigenvector:

$$VM = \alpha V \Rightarrow (VM)^T = M^T V^T = M V^T = \alpha V^T$$

and V is right eigenvector of M

Eigenvalues are same for left & right eigenvectors, since

$$\left.\begin{aligned} VM = \lambda V &\Rightarrow V(M - I\lambda) = 0 \\ MV = \lambda V &= (M - I\lambda)V = 0 \end{aligned}\right\}$$ Same characteristic equation $(M - I\lambda)$ for eigenvalues

Eigenvalues are:

$$\det\begin{pmatrix} 5-\lambda & 7 \\ 5 & 3-\lambda \end{pmatrix} = 15 - 8\lambda + \lambda^2 - 35 = \lambda^2 - 8\lambda - 20 = 0$$

$$(\lambda - 10)(\lambda + 2) = 0$$

$\lambda_1 = 1.0$

$\lambda_2 = -.2$

#7 11

```
//
//  P115B  HW1  #6
//  Program to compute Sum = a(1 + r + r^2 + r^3 ...+ r^n)
//
#include <iostream>
#include <fstream>
#include <math.h>
using namespace std;
int main()
{
    double a = 0;
    double r = 0.0;
    int n = 0;
    double series_sum = 0;
    cout << "Program to compute Sum = a(1 + r + r^2 + r^3 ...+ r^n)" << endl;
    cout << "Enter value for a ";
    cin >> a;
    cout << "Enter value for r ";
    cin >> r;
    cout << "Enter number of iterations ";
    cin >> n;
    for  ( int i = 0; i < n+1; i++)
    {
        series_sum  =  series_sum + pow(r,i);
    }
    series_sum  =  a * series_sum;
    cout << "Series sum =  ";
    cout <<  series_sum << endl;
    return 0;
}
********** OUTPUT  *********************
dave@dave-laptop:~/P115B_Fall_2011$ ./geom_series_sum.exe
Program to compute Sum = a(1 + r + r^2 + r^3 ...+ r^n)
Enter value for a 3
Enter value for r 2
Enter number of iterations 3
Series sum =  45
```

#8

```
//
//  P115B  HW1
//  Program to compute Fibonacci numbers
//
#include <iostream>
#include <fstream>
#include <math.h>
using namespace std;
int main()
{
    int lastValue = 1;
    int currentValue = 1;
    int nextValue = 0;
    double r = 0.0;
    int n = 0;
    double series_sum = 0;
    cout << "Program to compute Fibonacci numbers" << endl;
    cout << "Enter number of Fibonacci values to print"<< endl;
    cin >> n;
    cout << "First " << n << "  Fibonacci numbers are :" << endl;
    for  ( int i = 0; i < n; i++)
    {
        cout << currentValue << "   ";
        nextValue = currentValue + lastValue;
        lastValue = currentValue;
        currentValue = nextValue;
    }
    cout << endl;
    return 0;
}
```

12

```
********** OUTPUT *********************
dave@dave-laptop:~/P115B_Fall_2011$ ./fibonacci_numbers.exe
Program to compute Fibonacci numbers
Enter number of Fibonacci values to print
10
First 10  Fibonacci numbers are :
1  2  3  5  8  13  21  34  55  89
```