AM-8

$L_x \quad L_y$ $\quad\Updownarrow$ $\quad \hat{p}, \hat{x}$ $\quad\Updownarrow$

$$L_\pm = L_x \pm i L_y \qquad a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}\left(m\omega\hat{x} + i\hat{p}\right)$$

$$a =$$

$$[L_z, L_+] = [L_z, L_x + iL_y]$$

$$= i\hbar L_y + i(-i\hbar L_x) = \hbar(L_x + iL_y) = \hbar L_+$$

$$L_z L_+ = L_+ L_z + \hbar L_+$$

Similarly $[L_z, L_-] = -\hbar L_-$

$[L^2, L_\pm] = 0$ why?

* If f is an eigenfunct of L^2, L_z then so are $L_\pm f$

Proof $\quad L^2 L_+ f = L_+ L^2 f = L_+ \lambda f = \lambda L_+ f$

NB $\underline{\underline{\text{same}}}$ λ

$L_+ f = \mu f$

$$L_z L_+ f = (L_+ L_z + \hbar L_+) f$$

$$= L_+ \mu f + \hbar L_+ f$$

$$= (\mu + \hbar) L_+ f$$

Not same eigenvalue, $\hbar$ more

AM-9

Guess what about $L^2 L_- f \rightarrow \lambda L_- f$

$L_z L_- f \rightarrow (\mu - \hbar) L_- f$

Now there must be an f_{max} $L_+ f_{max} = 0$

Basically follows from fact that square of component of vector can never exceed total length^2 of vector,

$\mu \rightarrow \mu + \hbar, \mu + 2\hbar, \mu + 3\hbar$ $\Big\} \rightarrow$ eventually would exceed λ^2

which cannot be

$L_z f_{max} = \hbar l f_{max}$ $L^2 f_{max} = \lambda f_{max}$

$L_+ L_- = (L_x + iL_y)(L_x - iL_y)$

$= L_x^2 + L_y^2 - i(\underbrace{L_x L_y - L_y L_x}_{i\hbar L_z})$

$= L_x^2 + L_y^2 + \hbar L_z$

$L^2 = L_+ L_- + L_z^2 - \hbar L_z$ or

$L_- L_+ + L_z^2 + \hbar L_z$

$L^2 f_{max} = L_- L_+ f_{max} + L_z^2 f_{max} + \hbar L_z f_{max}$

$\quad\quad\quad\quad\quad\quad\; \downarrow \quad\quad\quad\quad\quad \downarrow \quad\quad\quad\quad \downarrow$

$\quad\quad\quad\quad\quad\quad\; 0 \quad\quad\quad\; \hbar^2 l^2 f_{max} \quad \hbar^2 l f_{max}$

$L^2 f_{max} = \hbar^2 l(l+1) f_{max}$ $\boxed{\lambda = \hbar^2 l(l+1)}$

AM-10

Can also analyze l_{min} (f_{min}):

$L_- f_{min} = \phi$

$L_z f_{min} = \hbar l' f_{min}$ $\quad L^2 f_{min} = \lambda f_{min}$

same λ as f_{max} why?

$$L^2 f_{min} = (L_+ L_- + L_z^2 - \hbar L_z) f_{min}$$

$$= [\phi + (l'\hbar)^2 - \hbar^2 l'] f_{min} = \hbar^2 l'(l'-1) f_{min}$$

$$\lambda = \underbrace{l'(l'-1)}_{} f_{min}$$

must $= l(l+1)$ $\Rightarrow$ $l' = -l$ or $l' = l+1$ X
because then f_{min} higher than f_{max}!

$\Rightarrow$ L_z eigenvalues $\{-l, -l+1, \ldots +l\}\hbar$ when

L^2 eigenvalue is $l(l+1)\hbar^2$

"I hope you're impressed" Griffiths

Note an oddity: If classical would expect max

component of a vector to be $\sqrt{|v|^2} = \sqrt{l(l+1)}\,\hbar$

One way to think about this is that we cannot

force L_y, L_x all the way to zero because it

would violate uncertainty rel'n demanded by $[L_x, L_z] \neq 0$.

(Griffiths problem 4.18)

$$L_+|l\,m\rangle = c\,|l,\,m+1\rangle$$

what is c? Ans

$$L_+|l\,m\rangle = \sqrt{l(l+1)-m(m+1)}\;|l\;m+1\rangle$$

plausible since for $m=l$ $\to \phi$

In accordance with $L_+|l\,m\rangle = \phi$

Likewise $L_-|l\,m\rangle = \sqrt{l(l+1)-m(m-1)}\;|l\;m-1\rangle$

Again if $m=-l$ $\to \phi$

IAM-12

More being impressed:

Mentioned QFT.
Note key trick of $L^2 f_{max}$, $L^2 f_{min}$ calc was using $L_+ f_{max} = L_- f_{min} = \phi$. (determined choice of which form of L^2 to use!)

QFT: $c^\dagger_{r_c} c_{r_b} \,|\, \vec{r}_a \, \vec{r}_b \rangle$

state with e^- at r_a r_b

$= c^\dagger_{r_c} c_{r_b} c^\dagger_{r_a} c^\dagger_{r_b} \,|\, \phi \rangle$

no electrons

move through

when $c_{r_b} |\phi\rangle = 0$

Guess? $|c^\dagger_{r_c} c^\dagger_{r_a}\rangle$!

$c^\dagger_{r_c} c_{r_b}$ is a "kinetic energy" which hops e^- from r_b to r_c.

AM-13

BRAKET LANGUAGE

$|\ell\, m\rangle$ $\qquad L^2|\ell\, m\rangle = \hbar^2 \ell(\ell+1)|\ell\, m\rangle$

More standard than f_ℓ^m of Griffiths $\qquad L_z|\ell\, m\rangle = \hbar m|\ell\, m\rangle$

m takes $2\ell+1$ values $-\ell$ to $+\ell$

Components in specific basis?

$|p\rangle$ $\qquad \hat{p}\,|p\rangle = p|p\rangle$

confusing?

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$$

Think about vectors

$\vec{v} = x\hat{x} + y\hat{y}$

"dual purpose" x

Answer for $|\ell\, m\rangle$

$$\langle r\,\theta\,\phi|\ell m\rangle = Y_{\ell m}(\theta,\phi)$$

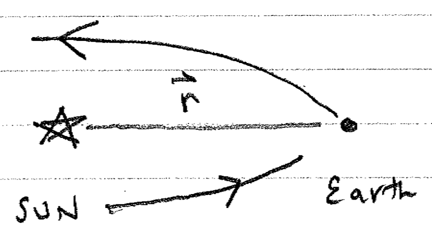
$\boxed{\boxed{\text{BUT}}}$ $\qquad 2\ell = $ integer is raising/lowering truncation condition

$\Rightarrow$ ℓ can be half integer!!

"Algebraic" calculation $\Rightarrow$ some extra solns. $\Rightarrow$ SPIN

As usual in physics math can lead to new understanding of nature!

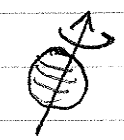
Classical Angular Momentum



$\vec{L} = \vec{r} \times \vec{p}$

from motion of Earths center of mass

But even if c of m stationary



$\vec{L}$ from "spin"

But in case of classical mechanics not a fundamental distinction. Why not? [Can "break earth up" and regard spin as "orbital" $\vec{L}$ of pieces, whose c of m <u>do</u> move]

In QM e^- carries spin AM in addition to orbital as it "goes around" H atom but its spin AM is "intrinsic": (As far as we know) <u>impossible</u> to break up e^-.