Assignment Five, Due Friday, November 2, 5:00 pm.

[1.] In understanding the behavior of spin-1/2 particles in quantum mechanics, you will encounter the $2 \times 2$ ‘Pauli matrices’

$$
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
$$

Compute the inverses of $\sigma_x, \sigma_y,$ and $\sigma_z$.

[2.] The ‘commutator’ of two matrices $A$ and $B$ is symbolized by $[A,B]$ and is defined by

$$
[A,B] = AB - BA
$$

Show that the ‘spin’ matrices

$$
S_x = \frac{\hbar}{2} \sigma_x, \quad S_y = \frac{\hbar}{2} \sigma_y, \quad S_z = \frac{\hbar}{2} \sigma_z
$$

obey

$$
[S_x, S_y] = i\hbar S_z, \quad [S_y, S_z] = i\hbar S_x, \quad [S_z, S_x] = i\hbar S_y
$$

[3.] to understand how spin wave functions evolve in time $t$ you will need the exponentials of the Pauli matrices,

$$
A = e^{-it\sigma_x}, \quad B = e^{-it\sigma_y}, \quad C = e^{-it\sigma_z}
$$

Using the definition

$$
e^M = I + M + \frac{1}{2}M^2 + \frac{1}{6}M^3 + \cdots
$$

compute $A, B$ and $C$.

[4.] In understanding the behavior of spin-1 particles in quantum mechanics, you will encounter the $3 \times 3$ matrices

$$
S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}
$$

Compute the commutators $[S_x, S_y], [S_y, S_z],$ and $[S_z, S_x]$ for spin-1. How do the results compare to the spin-1/2 case?