Short answers: 3 pts each (15 points)

a) Write down the general integral expression for finding the electric field from any continuous charge distribution at any point \( r \).

b) Write down the electric field from a very long straight wire with linear charge density \( \lambda \).

c) Write down the electric field from an infinite sheet of charge carrying charge density \( \sigma \).

d) Write down the electric field from a point charge \( q \).

e) Write down Coulomb’s law.
5 Problems (85 points)

Problem 1 (15 points)

A spherical shell of radius $a$ carries a uniform surface charge $Q$.

a) Write down Gauss’ Law and draw and label an appropriate Gaussian surface for this problem (3 points)

b) Write down 2 expressions that could be used to calculate the potential at a point $r$. (5 points)

c) Use one of these expressions to calculate the potential outside the spherical shell. (5 points)

d) What is the electric field inside the shell? Provide a reason for your answer. (2 points)
Problem 2. (15 points)

A Positive charge $Q$ is distributed along the positive $y$ axis between $y=0$ and $y=a$ with uniform charge density $\lambda$.

a) Find the $x$ and $y$ components of the electric field produced by this charge distribution on the point shown on the $x$ axis above, $(x,0)$. You can leave your answer in integral form. (10 points)

b) If we now place a point charge $-q$ at the point $(x,0)$, what are the $x$ and $y$ components of force at this point? (5 points)
Problem 3. (15 points)

In the previous problem, we were given a line charge positioned on the y-axis, above the horizontal. If we now add a line of opposite charge below the horizontal axis we can construct a crude model of an antenna as shown below. Assume the following about this charge distribution: the line of charge above the x-axis carries uniform charge density $+\lambda$ and extends from the origin to a point $(0,L/2)$ while the line of charge below the x-axis carries uniform charge density $-\lambda$ extends from the origin to $(0,-L/2)$.

What is another word that could be used to describe this charge distribution? (2 points)

Find the electric field at the point $(x,0)$ (8 points)
Problem 3 continued (5 points for part c)

c) In class we saw that the resulting electric field from this type of charge distribution falls off according to a $1/r^3$ behavior. Use the binomial expansion, along with the fact that $x$, the distance away from the charge distribution, is very large as compared to the length of the charge distribution, to prove this result. The binomial expansion formula is given below.

$$(x + a)^n = x^n + nx^{n-1}a + \cdots$$
Problem 4. (15 points)

A uniform electric field has magnitude \( E \) and is directed in the negative x-direction. The potential difference between point a, at \( x=0.6 \) and point b, at \( x=0.9 \) is 240V.

a) Which point, point a or point b, is at a higher potential? (justify your answer) (2 points)

b) Calculate the magnitude of the electric field (5 points)

c) A negative point charge \( q=-0.02 \) micro-coulombs, is moved from b to a. Calculate the work done. (8 points)
Problem 5. 25 points

A 2 microfarad capacitor is charged to 50 volts.

a) What is the charge on its plates? (5 points)

b) After charging, the capacitor is disconnected from the voltage source and is connected to another uncharged capacitor. The final voltage is 20 volts. What can you say about the sum of the charges on the two capacitors? (5 points)

c) What is the capacitance of the other (second) capacitor? (5 points)

d) How much energy is lost when the connection is made? (10 points)