2.68 - A rigid ball traveling in a straight line (the x-axis) hits a solid wall and suddenly rebounds during a brief instant. The $v_x$-t graph in Fig. P2.68 shows this ball’s velocity as a function of time. During the first 2.00 s of its motion, find (a) the total distance the ball moves and (b) its displacement. (c) Sketch a graph of $a_x$-t for this ball’s motion. (d) Is the graph shown really vertical at 5.00 s? Explain.

Figure P2.68

2.80 - Egg Drop. You are on the roof of the physics building, 46.0 m above the ground (Fig. P2.80). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on your professor’s head, where should the professor be when you release the egg? Assume that the egg is in free fall.

Figure P2.80

2.93 - During launches, rockets often discard unneeded parts. A certain rocket starts from rest on the launch pad and accelerates upward at a steady 3.00 m/s². When it is 235 m above the launch pad, it discards a used fuel canister by simply disconnecting it. Once it is disconnected, the only force acting on the canister is gravity (air resistance can be ignored). (a) How high is the rocket when the canister hits the launch pad, assuming that the rocket does not change its acceleration? (b) What total distance did the canister travel between its release and its crash onto the launch pad?
2.19. Figure 2.35 is a graph of the coordinate of a spider crawling along the x-axis. (a) Graph its velocity and acceleration as functions of time. (b) In a motion diagram (like Fig. 2.13b and 2.14b), show the position, velocity, and acceleration of the spider at the five times \( t = 2.5 \, s \), \( t = 10 \, s \), \( t = 20 \, s \), \( t = 30 \, s \), and \( t = 37.5 \, s \).

Figure 2.35 Exercise 2.19.

2.30. At \( t = 0 \) a car is stopped at a traffic light. When the light turns green, the car starts to speed up, and gains speed at a constant rate until it reaches a speed of 20 m/s 8 seconds after the light turns green. The car continues at a constant speed for 60 m. Then the driver sees a red light up ahead at the next intersection, and starts slowing down at a constant rate. The car stops at the light, 180 m from where it was at \( t = 0 \). (a) Draw accurate \( x-t \), \( v-t \), and \( a-t \) graphs for the motion of the car. (b) In a motion diagram (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of the car at 4 s after the light changes, while traveling at constant speed, and while slowing down.

2/4 The displacement of a particle which moves along the \( s \)-axis is given by \( s = (-2 + 3t)e^{-0.5t} \), where \( s \) is in meters and \( t \) is in seconds. Plot the displacement, velocity, and acceleration versus time for the first 20 seconds of motion. Determine the time at which the acceleration is zero.

2/30 A particle moving along the positive \( x \)-direction with an initial velocity of 12 m/s is subjected to a retarding force that gives it a negative acceleration which varies linearly with time for the first 4 seconds as shown. For the next 5 seconds the force is constant and the acceleration remains constant. Plot the velocity of the particle during the 9 seconds and specify its value at \( t = 4 \, s \). Also find the distance \( \Delta x \) traveled by the particle from its position at \( t = 0 \) to the point where it reverses its direction.
2.6 \[ x(t) = 1.5t^2 - 0.5t^3 + x_0 \]

a) \[ v [0 \rightarrow 2] = (x(2) - x(0))/(2 - 0) \]
\[ = (5.6 - 0)/(2 - 0) = 2.8 \text{ m/s} \]

b) \[ v [0 \rightarrow 4] = (x(4) - x(0))/(4 - 0) \]
\[ = (20.8 - 0)/(4 - 0) = 5.2 \text{ m/s} \]

c) \[ v [2 \rightarrow 4] = (x(4) - x(2))/(4 - 2) \]
\[ = (20.8 - 5.6)/(4 - 2) = 7.6 \text{ m/s} \]

2.10 a) \[ v = 0 \] when \( x(t) \) plot has zero slope.
Since \( v = \frac{dx}{dt} \). This is time IV

b) \[ v = \text{positive if slope positive} \]
\[ \text{v = constant if } x(t) \text{ is linear} \Rightarrow \text{time I} \]

c) \[ \text{time V} \]

d) \[ \text{Velocity is increasing in magnitude if velocity is positive} \]
\[ \text{and curve concave up or if velocity is negative and curve is concave down: II} \]

e) \[ \text{Likewise } v > 0 \] and concave down or \( v < 0 \) and concave up so III
2. \[ x(t) = 50 + 2t - 0.625t^2 \]

\[ a(t) = -1.125t \]

\[ \begin{align*}
  x(0) &= 50 \text{ cm} \\
  v(0) &= 2 \text{ cm/s} \\
  a(0) &= -0.125 \text{ cm/s}^2
\end{align*} \]

b) \[ v(t) = 0 = 2 - 1.125t \quad \rightarrow \quad t = 16 \text{ sec} \]

c) \[ x(t) = 50 = 50 + 2t - 0.625t^2 \]

\[ 2t - 0.625t = 0 \]

\[ t(2 - 0.625t) = 0 \quad \rightarrow \quad t = 0, \quad t = 3.2 \]

d) \[ x(t) = 60 = 50 + 2t - 0.625t^2 \]

\[ 60 = 50 + 10 \]

\[ t^2 - 3.2t + 160 = 0 \]

Could also check quadratic equation

\[ 40 = 50 - 10 \]

\[ t = \frac{1}{2} \left[ 3.2 \pm \sqrt{3.2^2 - 640} \right] = \frac{1}{2} \left[ 3.2 \pm 19.6 \right] = \begin{cases} 6.2 \text{ sec} \\ 25.8 \text{ sec} \end{cases} \]

\[ t = 6.2 \quad \Rightarrow \quad v = 2 - 1.125(6.2) = 1.22 \text{ cm/s} \]

\[ t = 25.8 \quad \Rightarrow \quad v = 2 - 1.125(25.8) = -1.22 \text{ cm/s} \]
3.

2-21

a) \( v^2 - v_0^2 = 2a(x-x_0) \)

\[ 45^2 - 0^2 = 2a(1.5) \]

\[ a = \frac{45^2}{3} = 675 \text{ m/s}^2 \]

b) \( v = v_0 + at \)

\[ 45 = 0 + 675t \quad t = 0.67 \text{ s} \]

**Cross-check:** \( x = x_0 + v_0t + \frac{1}{2}at^2 \)

\[ 45 = \frac{1}{2}(675)(0.67)^2 \quad \checkmark \]

2-42

\( y = y_0 + v_0t + \frac{1}{2}at^2 \)

\[ 0 = h + 0 + \frac{1}{2}(-9.8)(2.5)^2 \]

\[ h = \frac{1}{2}(9.8)(2.5)^2 = 30.6 \text{ m} \]

\( v = v_0 + at = 0 + (-9.8)(2.5) \)

\[ |v| = 24.5 \text{ m/s} \]

**Cross-check** \( (24.5)^2 - 0^2 = (2(-9.8))(0 - 30.6) \)

\[ 600 = 600 \quad \checkmark \]
4.

\[ y = v_0 + at \]
\[ t = 2.04 \text{ s} \]

\[ 20 = v_0 + (-9.8)t \]
\[ t = 6.12 \text{ s} \]

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
\[ 0 = 40 + \frac{1}{2}(9.8)t^2 \]
\[ 0 = t(40 - 4.9t) \Rightarrow t = 8.16 \text{ s} \]

\[ v = v_0 + (-9.8)t \]
\[ t = 4.08 \text{ s} \]

\[ v = 9.8 \text{ downwards at all times} \]

\[ t \]

\[ \text{max height} \]
\[ y - y_0 = v_0 t + \frac{1}{2} at^2 \]
\[ = 40(4.08) + \frac{1}{2}(-9.8)(4.08)^2 = 81.6 \]
5.

2-68
a) distance = area under v(t) curve

\[ \frac{1}{2} \times 30 \times 5 = 75 \text{ m} \]

In (5-20) interval distance = \[ \frac{1}{2} \times 20 \times 15 \]

= -150

TOTAL DISTANCE MOVED = 225 m

b) displacement = 75 - 150 = -75 m

\[ \begin{array}{c}
\text{a} \\
\text{m/s}^2 \\
\end{array} \\
\begin{array}{c}
\text{5} \\
\text{20} \\
\end{array} \\
\begin{array}{c}
\text{t} \\
\end{array} \\
\text{huge acceleration}
\]

2-80
Egg must fall vertically

\[ y = y_0 + v_0 t + \frac{1}{2} a t^2 \]

46 = 1.8 + 0 + \frac{1}{2}(-9.8) t^2

\[ t = 3 \text{ sec} \]

In this time, professor walks

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \]

\[ x - x_0 = 1.21(3) + 0 \]

= 3.6 m
2-93

Rocket speed

\[ \frac{V^2 - V_0^2}{2} = 2a(\gamma - \gamma_0) \]

When cannister disconnected

\[ V^2 - 0^2 = 2(3.3)(235) \]

\[ V = 39.4 \text{ m/s} \]

Cannister then falls

\[ y = y_0 + V_0 t + \frac{1}{2} at^2 \]

\[ 0 = 235 + 39.4t + \frac{1}{2} (-9.8)t^2 \]

\[ \text{Quadratic} \quad t = \frac{1}{9.8} \left[ 39.4 \pm \sqrt{(39.4)^2 - 4(-9.8)(235)} \right] \]

\[ = 12.03 \text{ s} \]

At this time rocket at

\[ y = y_0 + V_0 t + \frac{1}{2} at^2 \]

Time to reach 235

\[ 39.4 = 0 + 3.3t \]

\[ t = 11.9 \text{ sec} \]

b) Cannister takes

\[ 0 = 39.4 + (-9.8)t \]

\[ t = 4.02 \text{ s} \] to reach max height

Max height

\[ y = 235 + 39.4t + \frac{1}{2} (-9.8)(t^2) \]

\[ = 79.2 \text{ m} \]

Total dist travelled

\[ 2(79.2) + 235 = 393.4 \text{ m} \]
\[ x_s = 0 + 5t \]

a) \[ x_b = 40 + 0t + \frac{1}{2}(1.17)t^2 \]

\[ x_s = x_b \implies 5t = 40 + \frac{1}{2}(1.17)t^2 \]

\[ t = 9.55 \text{ sec} \]

b) \[ v_b = 0 + (1.17)(9.55) = 10.62 \text{ m/s} \]

c) [Graph showing positions of student and bus over time]

d) \[ v_{bus\ at\ 2nd\ soln} = 0 + (1.17)(49.27) \]

\[ = 83.38 \text{ m/s} \]

e) Redo a) with \( x_s = 3.5t \) \[ x_s = x_b \] has no soln (negative inside \( \sqrt{x} \) of quadratic eqn)
2-9.7 cont'd

\[ \begin{align*} f) \quad x_5 &= \frac{1}{2} (1.17) t^2 \quad x_b = 40 + \frac{1}{2} (1.17) t^2 \\
\end{align*} \]

\[ x_5 = x_b \]

\[ \frac{1}{2} (1.17) t^2 - V_5 t + 40 = 0 \]

\[ t = \frac{1}{2(\frac{1}{2})(1.17)} \left[ V_5 \pm \sqrt{V_5^2 - 4(40) \frac{1}{2}(1.17)} \right] \]

\[ \text{need to be } > 0 \]

\[ V_5^2 \geq 4(40) \frac{1}{2}(1.17) \Rightarrow V_5 \geq 3.7 \text{ m/s} \]

Student catches bus just as its velocity reaches there \[ 3.69 = 0 + 1.17 t \]

\[ t = \frac{3.69}{1.17} = 3.17 \]

Student runs \[ x_b = 3.69 (2.17) = 80 \text{ m} \]
**Exercise 2-19**

- **Initial Velocity**: $0.5\text{ m/s}$
- **Graphs showing motion over time**

**Exercise 2-30**

- **Graphs showing acceleration over time**

---

**First 8 sec**

- **Position**: $x = v_0 t + \frac{1}{2} a t^2$
- **Velocity**: $v = v_0 + at$
- **Acceleration**: $a = \frac{\Delta v}{\Delta t}$

- **For $t = 8$ sec**
  - $v = 0 + a(8)$
  - $a = \frac{v}{t}$

- **Final Position**
  - $x = x_0 + v_0 t + \frac{1}{2} a t^2$
  - $x = 0 + 0 + \frac{1}{2}(2.5)(8)^2 = 80\text{ m}$

**Time to go 60 m @ 20 m/s = 3 sec**

**Deceleration**

- $v^2 - v_0^2 = 2a(x - x_0)$
- $a = \frac{v^2 - v_0^2}{2(x - x_0)}$
- $a = -5\text{ m/s}^2$ (4 sec to decelerate from 20 m/s)
\[ s = (2 + 3t) e^{-t/2} \]

\[ v = \frac{ds}{dt} = 3e^{-t/2} + (-2 + 3t)e^{-t/2}(-\frac{1}{2}) \]

\[ = (4 - \frac{3}{2}t)e^{-t/2} \]

\[ a = \frac{dv}{dt} = -\frac{3}{2}e^{-t/2} + (4 - \frac{3}{2}t)e^{-t/2}(-\frac{1}{2}) \]

\[ = (-\frac{7}{2} + \frac{3}{2}t)e^{-t/2} \]

\[ a = 0 \quad \text{when} \quad \frac{3}{2}t = \frac{7}{2} \quad t = \frac{7}{3} \quad \text{sec} \]

\[ \begin{array}{c|c|c}
   t & s & \vphantom{\text{\text{}}} \\
   \hline
   0 & -2 & -2.000 \\
   1 & e^{-\frac{1}{2}} & 0.607 \\
   2 & 4e^{-1} & 1.476 \\
   3 & 7e^{-\frac{3}{2}} & 1.353 \\
   4 & 10e^{-2} & 1.079 \\
   6 & 16e^{-3} & 0.797 \\
   10 & 28e^{-5} & 0.181 \\
   20 & 58e^{-10} & 0.003 \\
   \end{array} \]
From E.102 text

\[ V_0 = 12 \text{ m/s} \]
\[ a = -\frac{3}{4} t \quad 0 < t < 4 \]
\[ a = -3 \quad 4 < t < 9 \]

\[ 0 < t < 4 \quad V = \int a(t) \, dt = \int -\frac{3}{4} t \, dt = -\frac{3}{8} t^2 + C \]
\[ C = 12 = V_0 \]

\[ V(t) = 12 - \frac{3}{8} t^2 \]
\[ V(4) = 12 - \frac{3}{8} (4)^2 = 6 \text{ m/s} \]

At \( t = 4 \)

\[ V = \int a(t) \, dt = \int -3 \, dt = -3t + C \]
\[ V(4) = 6 = C - 3(4) \quad \therefore C = 18 \]
\[ V(t) = 18 - 3t \]
\[ V = 0 \quad \text{at } t = 6 \]
\[ V = -9 \quad \text{at } t = 9 \]
\[
X = \int_0^4 \left(12 - \frac{3}{8} t^2 \right) dt + \int_4^6 \left(18 - 3 t \right) dt
\]

\[
= \left[ 12t - \frac{1}{8} t^3 \right]_0^4 + \left[ 18t - \frac{3}{2} t^2 \right]_4^6
\]

\[
= 48 - 8 + \frac{(108 - 54) - (72 - 24)}{54 - 48}
\]

\[
= 46 \text{ m}
\]