Theory for Quantum Gauge Glasses

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We develop a theory for the quantum gauge glass, with disorder in *both* the coupling strengths and the site energies. This model is closely related to XY magnets and bosons in random media. For properly chosen distributions of the site disorder we find two different glasses, Weak and Strong Glass regions, dominated by long range and local fluctuations, respectively. Strikingly, at the Strong Glass transition the non-linear susceptibility does not diverge. The Weak and Strong Glasses might be analogous to the two glassy phases observed in high T_c superconductors.

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A particularly active field of research these days is the quest to understand quantum phase transitions in the presence of disorder at T=0. Theoretical interest focussed on one dimensional transverse field Ising problems [1], on quantum critical behaviour of several spin models [2–4], and bosons in disordered media [5]. In one dimension scaling techniques delivered several exact results. In higher dimensions, after the original formulation [2] recently quantum fluctuations were incorporated as well [3,4]. While it was possible to describe the weakly disordered phases of the models satisfactorily, our understanding of the strongly disordered regions is still incomplete. Experimental studies intensified after the discovery of materials which are credible realizations of the spin-glass models, such as the $LiHoYF$ compounds, which exhibit Ising-type symmetry [6]. In this material at the low T quantum spin glass transition the non-linear susceptibility χ_{nl} did not diverge, a feature unexplained by recent theoretical studies [4,7].

In the present paper we develop a general formulation for the quantum spin glass prob-

lem, based on a *new bosonic field theory.* We identify *two glass transitions*, in agreement with previous approaches [4], but we can proceed further to explicitly calculate the critical behaviour even for strong disorder. We find that the *non-linear susceptibilty does not diverge at criticality.* The two glasses might be analogous to the two glassy phases, observed in high T_c materials at low and high magnetic fields.

Specifically we consider the quantum gauge glass model, which is equivalent to interacting bosons propagating in a magnetic field. This model possesses the right XY symmetry, frustration, and disorder, thus it is believed to contain the essential physics of vortices in Type II superconductors [8]. We consider an infinite range model as it allows for an exact treatment. (The corresponding classical model was investigated in Ref. [9].) At the same time it is thought to be equivalent to a mean field treatment of the realistic finite range physical problem. The infinite range formulation might be relevant for the $LiHoYF$ compounds as well, as the spins in these magnets are coupled by a dipolar term, i.e. the interaction is *long range.* In such cases the upper critical dimension is expected to be lower, thus mean field descriptions are more likely to provide a reasonable picture. Indeed, recent susceptibility measurements on the ordered parent compounds reveal a critical exponent $\gamma = 1$, i.e. the mean field value [10]. Furthermore it is our contention that an additional element of physics is crucial for these materials: the presence of a random component in the transverse magnetic field. Such a term can be motivated by observing that the transverse field induces a net transverse magnetization. The nonzero expectation value of the transverse component of a spin generates a random transverse field via the random interaction for the other spins it is coupled to. One can argue that such a randomness "smears out" the glass transition on the mean field level, in particular obliterating the singularity of the non - linear susceptibility, as seen in the experiments. The calculations in the present paper prove that this expectation is indeed true. While we demonstrated the absence of a singularity in an XY type glass model, since it is due to local fluctuations, we expect it to hold for the Ising case as well, which is relevant for the experiments on $LiHoYF$. Previous numerical approaches did not consider such terms, but the field theoretical methods of Ref. [4] represent this physics.

However it lead to runaway trajectories, preventing an analysis of the corresponding physics.

We consider the Hamiltonian:

$$H = -\sum_{i,j} J_{ij} a_i^\dagger a_j - \sum_i \mu_i a_i^\dagger a_i \tag{1}$$

where $J_{ij} = J_{ji}^*$ a complex random number for all $i < j$. Written in the form $J_{ij} = |J_{ij}| \exp(iA_{ij})$ exposes the magnetic field through the vector potential A_{ij}. Furthermore $< |J_{ij}|^2 >= J/N$, where N is the number of lattice sites. We set the energy scale by choosing $J = 1$. With this convention we not only have random phases along the bonds, representing the random fluxes, but also random bond strengths. As our results depend only on the second cumulant of the J_{ij}'s, the two models are expected to give the same results. $\mu_i = \mu + h_i$, where μ controls the density of the bosons, and h_i is a random site energy, with distribution P(h) over the *finite* support $[-\Delta, \Delta]$. Finally, for simplicity we consider hard core bosons, annihilated and created by a_i and $a_i^\dagger$. In the spin language these can be thought of as spin-1/2 spin-lowering and raising operators, while $a_i^\dagger a_i$ is directly related to the z component of the spin. According to this analogy, the Hamiltonian (1) describes an XY spin glass in a random transverse field.

At sufficiently high values of μ there will be exactly one boson per site leading to the formation of a "Mott-insulator" [5], which is characterized by a gap above the ground state and a lack of an order parameter. This corresponds to fully polarized spins in the z directon, i.e. it describes a paramagnetic state in the XY plane in the spin language. Lowering μ below some critical value μ_c generates holes in the system and presumably some glassy phases emerge. We will test these expectations by entering into the glassy regions from the high chemical potential direction.

In order to map out the phase diagram, we develop a many-body calculation for the system described by the Hamiltonian (1). The usual replica trick is employed, leading to the free energy $F = lim_{n\to 0} f_n/n$, where:

$$f_n = \frac{1}{2} Tr\ \mathbf{Q}^2 - \frac{1}{N} \sum_i \ \ln\ \Phi_i \tag{2}$$

$$\Phi_i = Tr\ e^{-\beta H_i^0} T_\tau \exp[\mathbf{a}^\dagger \mathbf{Q} \mathbf{a}] \tag{3}$$

Here the elements of the vector of annihilation operators $\mathbf{a}$ are indexed as $a_\alpha(\tau)$, and that of the matrix $\mathbf{Q}$ as $Q_{\alpha\beta}(\tau,\tau')$. The matrix product implies a summation over the replica indices and an integration over the time variables. Also, $H_i^0 = -\mu_i \sum_\alpha a_\alpha^\dagger a_\alpha$ and T_τ denotes the time ordering operator. In these equations $\mathbf{Q}$ satisfies the saddle-point condition: $Q_{\alpha\beta}(\tau,\tau') = \frac{1}{N}\sum_i \langle T_\tau a_\alpha^\dagger(\tau) a_\beta(\tau')\rangle_{\Phi_i}$. Here we introduce the key technical step of our work: a bosonic field theoretical formulation is developed as follows:

$$e^{\mathbf{a}^\dagger \mathbf{Q} \mathbf{a}} = \int \frac{D\varphi^* D\varphi}{Det\mathbf{Q}} \exp[-\varphi^* \mathbf{Q}^{-1} \varphi + \varphi \mathbf{a}^\dagger + \varphi^* \mathbf{a}] \tag{4}$$

This Hubbard-Stratonovich transformation is *exact*, because it has been performed *behind* the time ordering operator in Eq.(3), therefore the operators can be manipulated as c numbers. With the help of these new field variables the self-consistent equation for Q can be *exactly* rewritten as:

$$\begin{aligned} \mathbf{Q}(i\omega_n) &= \frac{1}{N}\sum_i \Big[\mathbf{E_i}(i\omega_n) - \mathbf{Q}(i\omega_n)\Big]^{-1} \\ \mathbf{E_i}(i\omega_n) &= [\mathbf{g_i^o}(i\omega_n) - \mathbf{\Sigma}_i(i\omega_n)]^{-1} \end{aligned} \tag{5}$$

The Fourier transformation diagonalized all matrices in frequency space, so from now on these remain matrices in the replica space only. Here $[\mathbf{g_i^o}]^{-1}(i\omega_n) = (\mu_i - i\omega_n)\delta_{\alpha\beta}$ is the on-site hole Green's function at site i. $\mathbf{\Sigma}_i(i\omega_n)$ is the self energy of the Green's function $G_i(i\omega_n)_{\alpha\beta} = \langle \varphi_\alpha^*(i\omega_n)\varphi_\beta(i\omega_n)\rangle_{S_i}$, with the local action:

$$S_i = -\varphi^* \mathbf{Q}^{-1} \varphi + \ln Tr e^{-\beta H_i^0} T_\tau e^{\varphi \mathbf{a}^\dagger + \varphi^* \mathbf{a}} \quad . \tag{6}$$

The φ field is *not* the soft-spin representation of the bosons, and thus no explicit constraint is required for its magnitude. In the spin language it is best thought of as a local magnetic field. We expand the action S_i up to fourth order, generating an effective φ^4 theory, the coefficients of which are determined by correlation functions of the hard core bosons. As now Wick's theorem applies, we can construct a perturbative expansion for the Green's function in order to determine the self energy $\mathbf{\Sigma}_i(i\omega_n)$. For now we stop at the level of the fully dressed Hartree diagram, which yields a replica diagonal self-energy. This is so

because the interaction is local in the replica space (Eq.(6)). In the absence of off-diagonal elements in the self energy it is reasonable to proceed with a replica symmetric form for $\mathbf{Q}$: $Q_{\alpha\beta}(i\omega_n) = \delta_{\alpha\beta}\sigma(i\omega_n) + \delta(i\omega_n)\beta q$. Substituting this form for $\mathbf{Q}$ in Eq.(5) determines these new quantities as:

$$\sigma(i\omega_n) = \frac{1}{N}\sum_i \frac{1}{E_i(i\omega_n) - \sigma(i\omega_n)} \tag{7}$$

$$q = \frac{q}{N}\sum_i \frac{1}{(E_i(0) - \sigma(0))^2} \tag{8}$$

We summarize the solutions of these equations at $T = 0$. Similar to the Bose glass problem [12], the asymptotics of the distribution of the site disorder plays a crucial role, because the first holes, being bosons, fill up the sites with the deepest energy levels. Spatial fluctuations are expected to renormalize the site-energy distribution $P(h)$ to a smooth function in finite dimensions. In our mean-field theory this does not happen by itself, so we take a smooth original distribution $P(h)$ with the asymptotics $\sim (\Delta - |h|)^r$. We find that the physics is the same for *any* $r > 1$: there are two different glassy regions with different critical exponents. To explore this physics, we take $r = 2$ in the rest of the paper.

In the disordered paramagnetic phase, characterized by $q = 0$, the self energy $\mathbf{\Sigma}_i(i\omega_n)$ follows an activated form because of the presence of the gap in the Mott-insulator, thus vanishing at $T = 0$. This gives $E_i(i\omega_n) = \mu_i - i\omega_n$ from Eq.(5) and the saddle-point equation (7) for σ takes the same form one would have gotten from an exact one-hole picture [18]. The density of states (DOS) can be obtained after an analytical continuation $i\omega_n \to \omega + i\eta$ from the imaginary part of σ. For weak disorder one recovers the results of Ref. [16]: a gap $\delta = \mu - \mu_c$ separates a band of states which starts as $\sim (\omega - \delta)^\zeta$ with $\zeta = 1/2$, the well known semicircular law of the random matrix theory of Wigner [13]. For strong disorder, i.e. $\Delta > \Delta_c = 1.393$, the right-hand side (*rhs.*) of Eq.(7) develops singular contributions at sites with the lowest site energies in the $\eta \to 0$ limit. As a result, ζ switches to 2, profoundly differing from the random matrix theory result.

At the critical point the gap closes and the average imaginary-time on-site Green's function $\langle a_i^\dagger(\tau) a_i\rangle$ decays as $\sim \tau^{\zeta+1}$. As we can see, the different values of ζ in the weak and

strong disorder regions result in different critical behaviours. Most importantly, the critical part of the non-linear susceptibility behaves as follows [15]:

$$\chi_{nl}^{-1} \sim 1 - frac1N \sum_i frac1(\mu_i - \sigma(0))^{.02} \tag{9}$$

. Comparing this expression to Eq.(8), and noting that $E_i(0) = \mu_i$, one can see that if the coefficient of q on the *rhs.* becomes 1, then the non-linear susceptibility diverges. For weak disorder the *rhs.* of Eq.(7) is analytic in the imaginary part of σ, therefore χ_{nl} diverges as the gap closes, (in fact with a 1/2 exponent) [16]. But in the strong disorder regime the same *rhs.* is non-analytic in Imσ, which happens before χ_{nl} diverges. This leads to the striking result that the non-linear susceptibility remains *finite* at the glass transition for strong disorder.

By now we have established two different critical regions upon exiting the Mott-insulating phase: we have found different critical behaviours for weak and strong disorder. Correspondingly, by lowering further the chemical potential the system enters into different glassy regions, what we call the Weak and the Strong Glass.

The Weak Glass phase is easiest understood by noticing that all formulae can in fact be expanded in the on-site disorder strength around $\Delta = 0$. In this case one finds $E = 2$, $\sigma(0) = 1$ and $\Sigma(0) \sim q \sim (\mu_c - \mu)$. Concerning the excitation spectrum, at low frequencies $E(i\omega_n) = 2 + const. \times (i\omega_n)$. Using this form in Eq.(***) reproduces the frequency of the density of states at criticality, i.e. the spectrum again starts with a square root form at low energies. Thus the Mott-to-Weak Glass transition is completely mean-field in character. In other words, the *global* order parameter q in Eq.(8) is generated by comparable contributions from all sites, indicating that the transition is driven by long range spatial correlations.

The physics of the Strong Glass transition is profoundly different. As the chemical potential is lowered, on the *rhs.* of Eq.(8), a few sites develop singular contributions. These *local* instabilities have to be regulated by generating a non-zero value for q. Obviously it is impossible to expand the *rhs.* around these singular points. This non-analyticity is well demonstrated in the rather non-traditional scenario for the transition: even *at the transition*

the coefficient of q on the *rhs.* of Eq.(8) is less than one. On the other hand, inside the glass phase (where $q > 0$) the coefficient of q must equal unity, therefore it *jumps* at the transition. This profound non-analyticity signals a genuinely new type of transition, which is driven by instabilities which are localized in space but extended in the imaginary time direction.

For the critical behaviour of q, the solution of the self-consistent equations gives $q \sim (\mu_c - \mu)^4$. Expanding the denominator for small frequencies gives $\mathrm{Im}\sigma \sim \omega^{1/2}$ for the spectrum. As we have shown, the spectrum is quadratic *at* the transition ($\zeta = 2$), i.e. the discontinuity in q's coefficient in Eq. (8) translates into a jump in the spectrum exponent, but the behaviour of q remains continuous, see Fig.1.

The following "phase diagram" emerges from these considerations. The transitions into the Weak and Strong Glass regions from the Mott phase are characterized by different exponents. The same is true approaching these boundaries from inside the glassy phases, however these two glasses *do not* possess different order parameters, hence they are not distinct phases in the thermodynamic sense. The spin glass order parameter q is non-zero in both, and the spectrum starts with a square root form in both as well.

When crossing from the Weak to the Strong Glass, the magnitude of q is a reliable indicator, as its value drops by orders of magnitude (see Fig.3). This is so because its asymptotic exponent of four is strongly different from the mean field value of one. This gives rise to an anomalously large critical region - the Strong Glass -, which is dominated by the local fluctuations, severely suppressing the value of q. When entering the Strong Glass from the Mott phase, the non-linear susceptibility does not diverge, neither does q develop an appreciable magnitude inside the glass. Here the best indicator is the low frequency spectrum. The Mott gap shrinks to zero at the transition. Entering the Strong Glass the decreasing chemical potential shifts the spectral weight further down. To accomodate this weight at low positive frequencies, the spectrum has to bulge up, taking a square root form: $\sim (\mu_c - \mu) \times \omega^{1/2}$, giving way to a shifted quadratic form only at higher frequencies (Fig.1). Thus measuring the DOS at a low fixed frequency assumes a value $\sim (\mu_c - \mu)$ inside the

Strong Glass.

Lastly we comment on the replica symmetry breaking. Substituting a general ultrametric $\mathbf{Q}$ into Eq.(5) one can show that all diagrams, which can cause replica symmetry breaking are proportional to the temperature, thus at $T = 0$ our replica symmetric solution is stable.

In sum, we have developed a theory for the quantum gauge glass, thought to be relevant for quantum spin glasses and bosons in disodered media. We included disorder *both* for the coupling strengths and for the site energies. Utilizing a new bosonic field theory we determined the phase diagram and the critical behaviour. We found Weak and Strong Glass regions, the physics of which is dominated by long range and local fluctuations, respectively. The most accessible experimental manifestation of the Strong Glass transition is the lack of divergence in the corresponding non-linear susceptibility at criticality. It is also tempting to draw analogy between our Weak and Strong glasses and the two glassy phases, observed in high T_c superconductors.

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