Magnetic susceptibility of exchange-disordered antiferromagnetic finite chains

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The low-temperature behavior of the static magnetic susceptibility $\chi(T)$ of exchange-disordered antiferromagnetic spin chains is investigated. It is shown that for a relatively small and even number of spins in the chain, two exchange distributions which are expected to occur in nanochains of P donors in silicon lead to qualitatively distinct behaviors of the low-temperature susceptibility. As a consequence, magnetic measurements might be useful to characterize whether a given sample meets the requirements compatible with Kane's original proposal for the exchange gates in a silicon-based quantum computer hardware. We also explore the dependence of $\chi(T)$ on the number of spins in the chain as it increases towards the thermodynamic limit, where any degree or distribution of disorder leads to the same low-temperature scaling behavior. We identify a crossover regime where the two distributions of disorder may not be clearly differentiated, but the characteristic scaling of the thermodynamic limit has not yet been reached.

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I. INTRODUCTION

Low dimensional quantum antiferromagnets have been intensively studied over the years, and became a reference problem in Condensed Matter Physics, Strongly Correlated Systems, and Statistical Mechanics.[1] Quantum spin chains may exhibit quantum phase transitions and, in the continuum limit, turned out to be an active area for application of quantum field theory in Condensed Matter Physics. Random quantum antiferromagnetic (AF) chains, which model the magnetic behavior of several quasi-one-dimensional compounds, have also attracted a great deal of attention.[2] Theoretical studies[3,4] of the quantum spin-1/2 disordered Heisenberg antiferromagnet in one dimension (1D) show that, for infinite chains, the magnetic susceptibility behavior is essentially insensitive to the specific disorder distribution. While the behavior in the thermodynamic limit is thus well established, there has not yet been a detailed study of how that limit is approached, and specifically how the behavior of the magnetic susceptibilities of finite chains might depend on the disorder distribution. This is an interesting question in light of current efforts towards nanoscale control and applications which, coupled to improved techniques of material growth, directed interest into magnetic nanostructures, in particular into finite magnetic chains.[5]

One of the most exciting potential applications of magnetism at the extreme microscopic level consists in utilizing the two-level dynamics of the electron spin as a physical implementation of the quantum-bit (qubit) in a solid state quantum computer, where the required entanglement between qubits could be provided by the exchange coupling between electrons.[6] For example, the quantum behavior of exchange-coupled electrons bound to an array of phosphorous donors in silicon is a key element in Kane's proposal for a Si-based quantum computer.[7] Low-temperature magnetic susceptibility measurements of P doped Si have provided valuable information regarding the exchange distributions in randomly three-dimensional (3D)-doped samples.[8] It is expected that similar measurements might also be relevant in characterizing linear arrays of donors. In Kane's proposal, two-qubit operations are mediated by the exchange interaction between electrons bound to nearest-neighbor P atoms in the chain. Previous studies have shown that this exchange coupling J is always AF, and that its strength is highly sensitive to the inter-donor relative positioning.[8–10] Indeed, changes of just one lattice parameter in the relative positioning of two P impurities may alter the magnitude of the coupling between them by orders of magnitude.[9,10] Controllable exchange coupling compatible with Kane's original proposal would be achieved if all donors in the chain could be positioned *exactly* along a single [100] crystal axis.[10] In this situation, the exchange coupling behavior is (as assumed by Kane[7]) similar to the hydrogenic case,[11] i.e., it decays exponentially with increasing interdonor distance. Uncertainties in the interdonor distances *along the chain* would result in a narrow distribution of values for J around a "target" value J_0. Assuming a substitutional donor positioning precision of about 1 nm in the Si lattice,[12] the perfect [100] alignment situation may be modeled by a trimodal exchange distribution $P_{tri}(J)$. However, if instead of perfect alignment the donor positions are randomly distributed among all substitutional sites within a small spherical region of 1 nm around the ideal ("target") impurity sites, the peculiar band structure of Si leads to a wide distribution of exchange coupling, peaked at $J=0$,[10] causing difficulties in the operation and control of the "exchange gates." We approximate such distribution here by an exponential function, $P_{exp}(J)$ for $J>0$. It is, therefore, clear that the exchange disorder distribution is also highly sensitive to the positioning distribution of the P donors in Si.

The magnetic susceptibility behavior of *infinite chains* is essentially insensitive to the specific disorder distribution, as mentioned above, indicating that susceptibility measurements would not differentiate among the two cases mentioned above in this limit. Of course this may be different for *finite chains*, a situation which is also of practical interest in terms of guiding current fabrication efforts towards P donors positioning in Si.[12,13] The aim of the present work is to shed light on this problem by investigating the relation between the magnetic response of linear chains of spins and the distributions of the exchange interaction within the chains. Our results indicate that for AF disordered chains with even and relatively small number N of sites, the exchange distributions $P_{tri}(J)$ and $P_{exp}(J)$ lead to quite distinct low temperature behavior of the zero-frequency uniform magnetic susceptibility. Hence, the two distributions could be experimentally distinguished by sufficiently sensitive magnetic measurements, providing useful information regarding donor alignment.

This paper is organized as follows. In Sec. II we briefly review results available in the literature regarding infinite AF chains. In Sec. III we consider finite chains with relatively small number of spins, starting from spin pairs and trios for which analytical solutions are obtained, as well as eight-spin chains, which are solved numerically. In Sec. IV we analyze the crossover into the thermodynamic limit by solving for longer chains via a quantum Monte Carlo method. Our summary and conclusions are presented in Sec. V.

II. INFINITE ANTIFERROMAGNETIC CHAINS

The Hamiltonian describing an open chain with N spins is

$$\mathcal{H} = \sum_{i}^{N-1} J_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}, \tag{1}$$

where the spin quantum number is $S=1/2$. Since we are interested in AF chains, we assume that $J_i \geqslant 0$ for every i. We briefly review in this section several pertinent results available in the literature for AF chains in the $N\to\infty$ limit.

The ground state of an *ordered* infinite chain ($J_i=J$ for every i) can be obtained from Bethe ansatz. Griffiths[14] has shown that the zero temperature susceptibility per spin of such system is finite and given by $\chi(T=0)/\chi_0(J)=1/\pi^2$, where $\chi_0(J)=g^2(\mu_B)^2/J$, where μ_B is the Bohr magneton and $g(=2)$ is the Landé factor. For general T, field theory methods [$k=1$ Wess-Zumino-Witten (WZW) nonlinear σ model][15] give

$$\frac{\chi(T)}{\chi_0(J)} = \frac{1}{(\pi)^2}\left(1 + \frac{1}{2\ln(T_0/T)}\right), \tag{2}$$

where T_0 is a temperature cutoff. Quantum Monte Carlo (QMC) calculations of $\chi(T)$ have been carried out by Kim *et al.*[16] and their results for $T_0/J=1.8$ (we use units of energy for temperature, that is, the Boltzmann constant k_B is set equal to one) are well fitted by the WZW expression (2).

According to real-space renormalization group theory,[4] the introduction of any amount of disorder drives the system into a random singlet phase, in which each spin forms a singlet pair with another spin; pairs with arbitrarily long distance also exist. Bonds among distant spins, however, correspond to very weak coupling. The low-temperature excitations basically involve breaking these weakest bonds, resulting in nearly-free spins giving rise to a Curie susceptibility modified by the statistics in the number of contributing spins. As a result the magnetic susceptibility at low T diverges as[4]

$$\chi(T\to 0) \sim 1/[T(\log T)^2]. \tag{3}$$

III. FINITE CHAINS

The above results refer to chains in the thermodynamic limit ($N\to\infty$). For *finite ordered* rings (periodic boundary conditions), Bonner and Fisher[17] have calculated the susceptibility per spin $\chi_N(T)$ for up to $N=11$ spins based on direct diagonalization of $\mathcal{H}$ in the presence of a magnetic field. They have found that $\chi_N(T\to 0)$ exhibits distinct behavior depending on whether N is even or odd. In the first case, pairs of neighboring spins tend to form singlets and $\chi_{N=even}(T\to 0)\to 0$, whereas in the second case, the occurrence of unpaired spins leads to a Curie-law behavior $\chi_{N=odd}(T\to 0)\sim 1/T\to\infty$. These results immediately raise the question as to what extent the behavior of $\chi_N(T)$ changes by the introduction of disorder. Moreover, could we infer, based on the magnetic response, the type of exchange disorder distribution? The relevant distributions here are: (i) Trimodal, with

$$P_{tri}(J) = \left(\frac{1}{3}\right) \times\{\delta(J-J_0) + \delta(J-(1+W)J_0) + \delta(J-(1-W)J_0)\}, \tag{4}$$

where J_0 and W are both positive, with $W<1$, and (ii) exponential, with

$$P_{exp}(J) = \frac{1}{J_0}e^{-J/J_0}\Theta(J), \tag{5}$$

where Θ is the step-function. Note that in both cases $\langle J\rangle=J_0$, the exchange "target" value.

We consider initially the $N=2$ case, for which the susceptibility per spin is given by[18]

$$\frac{\chi_2(T,J)}{\chi_0(J)} = \frac{\beta J}{3+e^{\beta J}}, \tag{6}$$

where $\beta=1/T$ and $\chi_0(J)$ is given above Eq. (2). Considering the average of χ_2 over the above distributions, it is clear that $\langle\chi_2\rangle_{tri}=\int_0^\infty P_{tri}(J)\chi_2(T,J)dJ$ vanishes as $T\to 0$, as in the absence of disorder. For the exponential distribution, it is convenient to split the integral for $\langle\chi_2\rangle_{exp}$ into two terms

$$\langle\chi_2\rangle_{exp} = \frac{\beta}{J_0}\int_0^{\alpha/\beta}\frac{e^{-J/J_0}}{3+e^{\beta J}}dJ + \frac{\beta}{J_0}\int_{\alpha/\beta}^{\infty}\frac{e^{-J/J_0}}{3+e^{\beta J}}dJ, \tag{7}$$

where α is a constant, and $\langle\chi_2\rangle_{exp}$ is given in units of $(g\mu_B)^2$. For sufficiently low temperatures, α can be chosen such that $\alpha/\beta \ll J_0$ and $e^{-\alpha}\ll 1$. Hence, in the first integral (corresponding to small values of J), we can approximate $e^{-J/J_0}\approx 1$, while in the second one, the term $e^{\beta J}$ is always much greater than 1, leading to

$$J_0\langle\chi_2\rangle_{exp} \approx C_1 + e^{-\alpha} \approx C_1, \tag{8}$$

where $C_1=\int_0^{\alpha}1/(3+e^x)dx \approx \int_0^{\infty}1/(3+e^x)dx=\ln(4)/3$. Thus, as $T\to 0$, $\langle\chi_2\rangle_{exp}$ approaches a nonzero value, in contrast with the trimodal distribution result.

For an open $N=3$ chain, the susceptibility per spin is[18]

$$\chi_3(T,J_1,J_2) = (\beta/12)\frac{5+e^{\beta(J_1+J_2)/2}\cosh((\beta/2)\sqrt{J_1^2-J_1J_2+J_2^2})}{1+e^{\beta(J_1+J_2)/2}\cosh((\beta/2)\sqrt{J_1^2-J_1J_2+J_2^2})}. \tag{9}$$

It is clear that for non-negative values of J_1 and J_2, χ_3 exhibits a Curie-type divergence as $T\to 0$. As a consequence, the average of χ_3 over the trimodal distribution, $\langle\chi_3\rangle_{tri}$, also diverges as T approaches 0. Regarding the exponential distribution, the change of variables $J_1=J\sin\theta$ and $J_2=J\cos\theta$ leads to

$$J_0^2\langle\chi_3\rangle_{exp} = (\beta/12)\int_0^{\pi/2}d\theta\int_0^{\infty}e^{-(\sin\theta+\cos\theta)J/J_0}\frac{5+f(\beta J,\theta)}{1+f(\beta J,\theta)}JdJ, \tag{10}$$

where $f(x,\theta)=e^{x(\sin\theta+\cos\theta)/2}\cosh((x/2)\sqrt{1-\sin\theta\cos\theta})$. As for the $N=2$ case, we split the integral above into two, which we label I_1 and I_2, corresponding to $0\leqslant J\leqslant\alpha/\beta$ and $\alpha/\beta\leqslant J\leqslant\infty$, respectively. For sufficiently low T, α is again chosen such that $\alpha/\beta \ll J_0$ and $e^{-\alpha}\ll 1$. Since $1\leqslant\sin\theta+\cos\theta\leqslant\sqrt{2}$, the term $e^{-(\sin\theta+\cos\theta)J/J_0}$ in the first integral (corresponding to small values of J) can be approximated by 1, giving

$$I_1 \approx \frac{1}{\beta^2}\int_0^{\pi/2}d\theta\int_0^{\alpha}\frac{5+f(x,\theta)}{1+f(x,\theta)}xdx = C_1'/\beta^2, \tag{11}$$

where C_1' is T-independent. In the second integral, since $\beta J \geqslant \alpha$ and α is large, the T-dependent term in both the numerator and denominator of the integrand is much larger than 1, so that

$$I_2 \approx \int_0^{\pi/2}d\theta\int_{\alpha/\beta}^{\infty}e^{-(\sin\theta+\cos\theta)J/J_0}JdJ$$
$$\approx \int_0^{\pi/2}d\theta\int_0^{\infty}e^{-(\sin\theta+\cos\theta)J/J_0}JdJ = C_2', \tag{12}$$

where C_2' is T-independent. Thus, from Eq. (10), $J_0\langle\chi_3\rangle_{exp} = (\beta/12)(I_1+I_2)/J_0 \approx \beta C_2'/J_0$, which diverges following a Curie law as $T\to 0$.

For a better physical insight, it is instructive to compare the $N=2$ and $N=3$ cases. We note that in the low-T regime, the T-dependent behavior of χ_N is dominated in the first case by the disorder distribution, while in the second by the odd parity of N. The Curie-type low-T behavior of $\langle\chi_3(T)\rangle$ is related to the occurrence of "unpaired" spins, independently of $P(J)$. It is interesting that the large-J tail of the distribution gives the dominant contribution to the average behavior, namely the integral given by I_2 in Eq. (12), corresponding to the more strongly coupled three-spin chains. On the other hand, for $N=2$, the behavior of $\langle\chi_2(T)\rangle$ at low T is dominated by the contribution from small values of J (more precisely, from values of $J \ll \min\{T,J_0\}$), as shown in Eqs. (7) and (8). The occurrence of arbitrarily small values of J in the exponential distribution weakens the tendency of neighboring spins to form singlets,[17] leading to a finite value of $\langle\chi_2(T=0)\rangle_{exp}$, while $\langle\chi_2(T=0)\rangle_{tri}=0$.

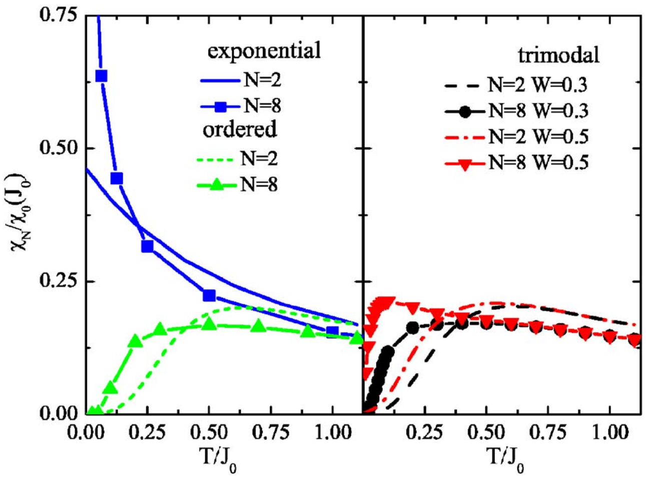

FIG. 1. (Color online) Low-temperature behavior of the magnetic susceptibility per spin of disordered antiferromagnetic spin chains with exponential and trimodal distributions of exchange coupling. The plots give $\chi_N/\chi_0(J_0)$ vs T/J_0 for $N=2$ and 8. Results for the exponential disorder distribution and for ordered chains are given on the left panel, whereas those for the trimodal disorder distributions with width $W=0.3$ and 0.5 are given on the right panel. All results are for open boundary conditions. For $N=8$ the statistical error bars are smaller than the data points, and the straight lines joining the points are just guides to the eye.

We expect the above considerations to apply to other finite values of N. As odd-parity N chains always lead to unpaired spins, and thus to a Curie-type low-T divergence of $\langle\chi_{N=odd}(T)\rangle$, susceptibility measurements would not be useful for distinguishing among different exchange disorder distributions if N is odd. Since analytical calculations become impractical as N increases, in order to illustrate the even-parity case we have carried out numerical calculations for both $\langle\chi_8\rangle_{tri}$ and $\langle\chi_8\rangle_{exp}$ as a function of T. We obtain the susceptibility numerically by determining the spin-spin correlation function $\langle S_i^z S_j^z\rangle$ from which, through the fluctuation-dissipation theorem, we obtain $\chi=\beta\Sigma_{i,j}\langle S_i^z S_j^z\rangle$. For each temperature we have averaged over 10 000 realizations of disordered configurations. For the trimodal distribution, two values for the width parameter have been considered, namely $W=0.3$ and 0.5. Results are presented in Fig. 1, which shows susceptibility curves for chains with $N=2$ and 8, for trimodal and exponential disorder distributions, as well as for the ordered chain cases. We note that results for $\langle\chi_N(T)\rangle_{tri}$ are qualitatively very similar, regardless of the width parameter W, even in the limit $W=0$, corresponding to the ordered chains. All curves reach a maximum and eventually decrease as $T\to 0$, going to zero for $T=0$. We remark the obvious fact, also illustrated in Fig. 1, that as W increases the maximum in χ and the sharp downturn toward zero value occur at lower

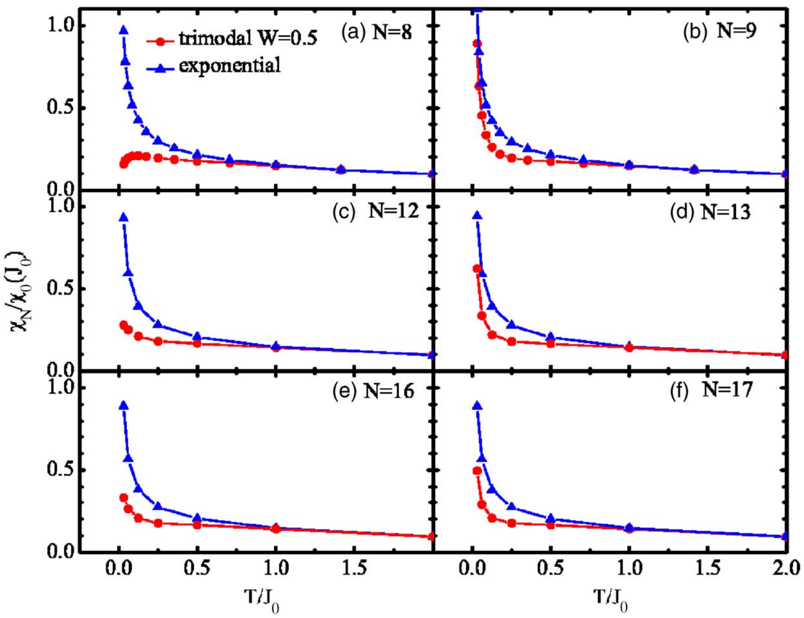

FIG. 2. (Color online) Magnetic susceptibility per spin for disordered finite chains of N spins for trimodal ($W=0.5$) and exponential disorder distributions. For $T\to 0$ the $N=8$ results for the two distributions are clearly distinct, while for $N=9$ they are practically collapsed. As the number of spins in the chain increases, the results for the two distributions and N_{even} (left column) approach each other, as results for N_{even} and $N_{even}+1$ chains (successive rows) also approach each other. Lines are guides to the eye and statistical error bars are smaller than the data points.

temperatures. The results for the exponential distribution are markedly different, with an increasing $\langle\chi_N(T)\rangle_{exp}$ for decreasing T. In principle the trimodal and exponential distributions might be identified and differentiated through low-temperature susceptibility measurements in such even-parity chains.

IV. CROSSOVER REGIME AND THERMODYNAMIC LIMIT

Further increase in N eventually leads to the thermodynamic limit behavior[19] given in Eq. (3). For even N, approach to such behavior, giving a divergent χ as $T\to 0$, might be expected from the comparison between the results of $\langle\chi_2(T)\rangle_{exp}$ and $\langle\chi_8(T)\rangle_{exp}$ in Fig. 1, but it is not so clear for the trimodal distributions. Another puzzling point regards the sensitivity to even (N_{even}) or odd (N_{odd}) values of N. For small-N_{odd}, χ exhibits a Curie-type $\sim 1/T$ divergence at $T\to 0$ independent of the disorder distribution, as discussed for $N=3$ in Sec. III, while small-N_{even} chains are quite sensitive to $P(J)$. But odd- and even-chains results must approach each other, possibly with some sensitivity to the type of disorder (remanent from N_{even}), as N increases.

In order to clarify these points we have calculated the susceptibility for larger values of N using a stochastic series expansion (SSE) method. The SSE is a QMC method based on the Taylor expansion of the Boltzmann weight operator $e^{-\beta\mathcal{H}}$ up to a very high order.[20] The partition function and observables can then be evaluated via importance sampling of the different terms appearing in this series. Choosing a large enough order for the expansion, the systematic errors introduced by the truncation of the series are negligible. The SSE method also allows us to use importance sampling update schemes based on global changes of the system (cluster or loop updates) that are extremely efficient, especially for the highly symmetric Heisenberg model studied here. We have adjusted the precision of the data obtained through importance sampling so that the errors obtained on each individual realization are roughly one order of magnitude smaller than a typical difference between two realizations. We have considered open chains and averaged our results over 10 000 disorder realizations (except for the large size $N=128$ where we used periodic boundary conditions and averaged over only 100 realizations).

We present in Fig. 2 results for $\langle\chi_N(T)\rangle_{tri}$ and $\langle\chi_N(T)\rangle_{exp}$ for increasing N up to $N=17$ for the exponential and trimodal ($W=0.5$) distributions. We identify here the following trends towards the thermodynamic limit: (i) From the frames for N_{even} (on the left) we see that the well differentiated disorder distribution results for $N=8$ in Fig. 2(a) approach each other as N_{even} increases [Figs. 2(c) and 2(e)]; (ii) from the frames for N_{odd} (on the right) we do not identify significative changes as the Curie behavior discussed in Sec. III for $N=3$ acquires logarithmic corrections [eventually leading to the asymptotic behavior governed by Eq. (3)] which are not easily captured on this scale and through this range of N_{odd} values; (iii) following successive rows we see that the even-odd differences (namely going from N_{even} to $N_{even}+1$) become less prominent as N_{even} increases Figs. 2(a)–2(f). From the practical point of view, it is clear from (i) that, contrary to the $N=8$ results, for $N=16$ the two distributions lead to very similar qualitative types of behavior of the susceptibility, indicating that, for the particular distributions and temperature range considered here, they may not be clearly differentiated beyond this size of chains, regardless of the parity of N.

Approach to the thermodynamic limit, given in Eq. (3), is usually investigated by plotting $(T\chi)^{-1/2}$ versus $\log T/J_0$, which leads to a linear behavior at low temperatures in this limit.[19] In Fig. 3 we illustrate the crossover regime by presenting such scaled plots for $N=16$ and 128 for trimodal ($W=0.5$) and exponential distributions. In all cases, straight lines (dashed for trimodal and solid for exponential) are drawn through the two lowest-T calculated points. We note that for the trimodal distribution and $N=16$ such line does not include any other calculated point, whereas for $N=128$ it

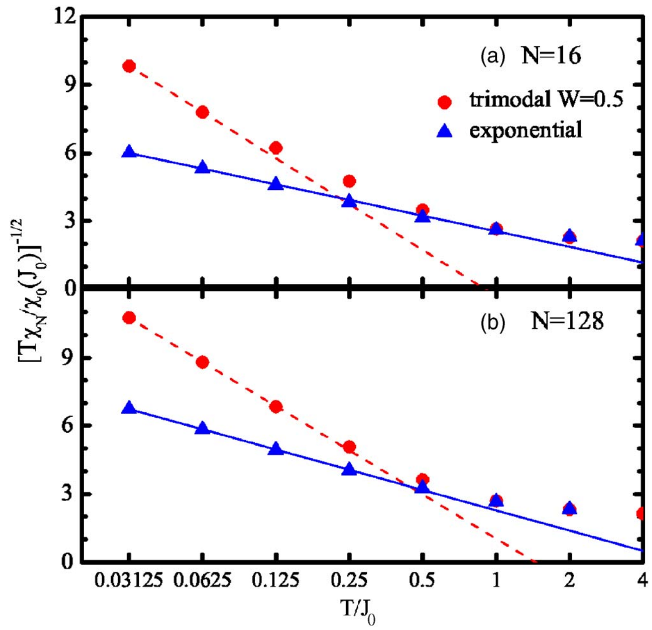

FIG. 3. (Color online) Scaled plots for the low-temperature behavior of the magnetic susceptibility per spin of disordered chains, illustrating the approach to the thermodynamic limit as N increases. The plots give $1/\sqrt{T\chi_N/\chi_0(J_0)}$ vs T/J_0 (logarithm scale) for exponential and trimodal ($W=0.5$) distributions in chains with $N=16$ and 128. The thermodynamic limit ($N\to\infty$) corresponds to the linear behavior indicated by the straight solid and dashed lines drawn to connect the two lowest-T data points for exponential and trimodal distributions, respectively.

gives a good fitting for T/J_0 up to about 0.25, indicating that the thermodynamic limit has been reached up to this temperature for this value of N. For the exponential distribution, the approach to the thermodynamic limit scaling with increasing N is faster, although careful analysis of the data plotted in Fig. 3 shows that the points for $N=16$ and exponential distribution [triangles in Fig. 3(a)] down-turn from the solid line at small T, and the good agreement for $T/J_0=1$ in (a) is fortuitous, as it does not remain for $N=128$ [triangles in Fig. 3(b)]. It is interesting to note that the solid (dashed) line in Fig. 3(a) is nearly parallel to the solid (dashed) line in Fig. 3(b), indicating that some aspects of the $N\to\infty$ behavior are already captured at the smaller N values at low-T. Figure 3(b) shows that in the thermodynamic limit the susceptibility is quantitatively quite sensitive to the disorder distribution, although in practice this is probably not as valuable a tool to identify the disorder distribution as the differences encountered for the smaller even values of N [e.g., Fig. 2(a)].

V. SUMMARY AND CONCLUSIONS

We have investigated the low-T behavior of the uniform magnetic susceptibility of exchange disordered spin-1/2 AF chains as a function of the number N of spins in the chain and disorder, for trimodal and exponential disorder distributions. Formally, the key distinction among the two distributions considered here is that the exponential distribution is not bound, so that pair exchange coupling J arbitrarily close to zero may occur, while the trimodal distribution is bound and J does not become arbitrarily small.

For chains with even and relatively small number of spins ($N_{even}\lesssim 8$) the susceptibility displays distinct behaviors for the two exchange distributions which are expected to occur in nanochains of P donors in Si. According to our results in Fig. 1, it might be possible to identify the atomic-scale positioning of the P atoms in such chains, thus providing complementary information on whether sample preparation techniques meet the requirements compatible with Kane's original proposal for a quantum computer hardware. For larger values of N ($N\gtrsim 16$) such differentiation would not be so straightforward, as illustrated in Fig. 2. We also note that $\chi_N(T)$ becomes less sensitive to N being even or odd when $N\gtrsim 16$. For completeness, we have also investigated the approach to the thermodynamic limit, and we have found that for $N=128$ the expected scaling of $\chi(T\to 0)$ is already obtained, while for $N=16$ the system is still far from the thermodynamic limit, as discussed in Fig. 3.

We, therefore, identify three regimes as N increases: (i) When $N\lesssim 8$ the even-N disordered chains present quite distinct behaviors according to the exchange distribution; (ii) for intermediate N values, illustrated here by $N=16$, the low temperature behavior of the magnetic susceptibility corresponding to different distributions is not easily differentiated, although the thermodynamic limit has not yet been reached; (iii) for larger N, illustrated here by $N=128$, the two distributions follow the thermodynamic limit scaling, which is a signature that the random singlet phase is formed at the lowest T.

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[1] E. H. Lieb and D. C. Mattis, *Mathematical Physics in One Dimension* (Academic, New York, 1966); J. Bernasconi and T. Schnieder, *Physics in One Dimension* (Springer-Verlag, New York, 1981).
[2] M. Steiner, J. Villain, and C. Windsor, Adv. Phys. **25**, 87 (1976).
[3] S. K. Ma, C. Dasgupta, and C. K. Hu, Phys. Rev. Lett. **43**, 1434 (1979); C. Dasgupta and S. K. Ma, Phys. Rev. B **22**, 1305

(1980).
[4] D. S. Fisher, Phys. Rev. B **50**, 3799 (1994).
[5] P. Gambardella, A. Dallmeyer, K. Maiti, M. C. Malagoli, W. Eberhardt, K. Kern, and C. Carbone, Nature (London) **416**, 301 (2002).
[6] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[7] B. E. Kane, Nature (London) **393**, 133 (1998).
[8] K. Andres, R. N. Bhatt, P. Goalwin, T. M. Rice, and R. E. Walstedt, Phys. Rev. B **24**, 244 (1981).
[9] B. Koiller, X. Hu, and S. Das Sarma, Phys. Rev. Lett. **88**, 027903 (2002).
[10] B. Koiller and X. Hu, IEEE Trans. Nanotechnol. **4**, 1536 (2005).
[11] C. Herring and M. Flicker, Phys. Rev. **134**, A362 (1964).
[12] S. R. Schofield, N. J. Curson, M. Y. Simmons, F. J. Rueß, T. Hallam, L. Oberbeck, and R. G. Clark, Phys. Rev. Lett. **91**, 136104 (2003).
[13] T. Schenkel, A. Persaud, S. J. Park, J. Nilsson, J. Bokor, J. A. Liddle, R. Keller, D. H. Schneider, D. W. Cheng, and D. E. Humphries, J. Appl. Phys. **94**, 7017 (2003).
[14] R. B. Griffiths, Phys. Rev. **135**, A659 (1964).
[15] S. Eggert, I. Affleck, and M. Takahashi, Phys. Rev. Lett. **73**, 332 (1994).
[16] Y. J. Kim, M. Greven, U.-J. Wiese, and R. J. Birgenau, Eur. Phys. J. B **4**, 291 (1998).
[17] J. C. Bonner and M. E. Fisher, Phys. Rev. **135**, A640 (1964).
[18] J. T. Haraldsen, T. Barnes, and J. L. Musfeldt, Phys. Rev. B **71**, 064403 (2005).
[19] S. Todo, K. Kato, and H. Takayama, *Computer Simulation Studies in Condensed Matter Physics XI*, edited by D. P. Landau and H.-B. Schüttler (Springer-Verlag, Berlin, 1999), pp. 57–61; Also at cond-mat/9803088.
[20] A. W. Sandvik, Phys. Rev. B **59**, R14157 (1999).