NAME: KEY

# Midterm Exam, Physics 9C, Winter 2016

**General Instructions:** You are allowed one (double sided) note paper. Please show all your work, and give units for all answers and on all graphs. Credit will only be given for complete solutions.

**1.** (30 points) (a) What information does the operation "gradient" tell you about a scalar function $f(x, y, z)$? What is $\vec{\nabla} f$ for $f(x, y, z) = 5xy + y^2 + xyz$?

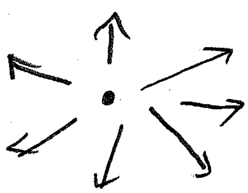
The gradient points in the direction of maximal increase of f.

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= (5y + yz)\hat{x} + (5x + 2y + xz)\hat{y} + xy\,\hat{z}$$

(b) What information does the operation "divergence" tell you about a vector function $\vec{E}(x, y, z)$? Draw a collection of vectors and label a point where the divergence would be big. What is $\vec{\nabla} \cdot \vec{E}$ for $\vec{E} = x^2\,\hat{\imath} + y^2\,\hat{\jmath} + z^2\,\hat{k}$?

The divergence measures vectors spreading apart.



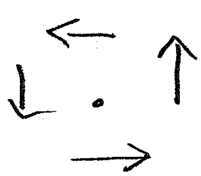
$$\vec{\nabla} \cdot \vec{E} = \frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z$$

$$= 2x + 2y + 2z$$

Large divergence at •

(c) What information does the operation "curl" tell you about a vector function $\vec{B}(x, y, z)$? Draw a collection of vectors and label a point where the curl would be big. What is $\vec{\nabla} \times \vec{B}$ for $\vec{B} = -y\,\hat{\imath} + x\,\hat{\jmath} + 3\,\hat{k}$?

The curl measures the rotation of vectors around a point

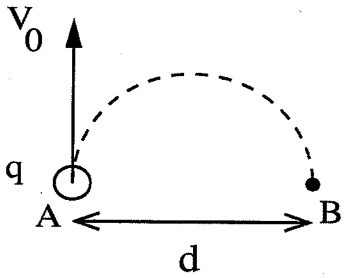


$$\vec{\nabla} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & 3 \end{vmatrix}$$

$$= 0\,\hat{x} + 0\,\hat{y} + 2\,\hat{z}$$

large curl at •

**2.** (20 points) A charge $q = 4 \times 10^{-20} C$ has mass $m = 3 \times 10^{-26} kg$ and speed $v_0 = 2 \times 10^6 m/s$. Find the magnitude and direction of the magnetic field that will cause the charge to move in a semi-circular path from $A$ to $B$ if the diameter $d = 0.50m$. (b) Find the time required to move from $A$ to $B$.




a) $F = ma$

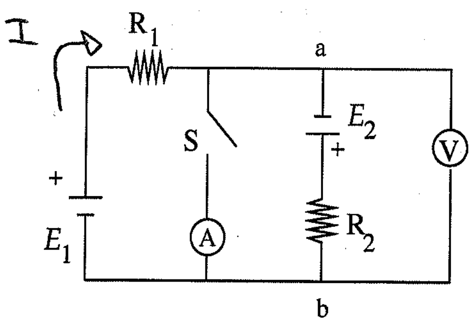
$qvB = mv^2/r$

Motion is circular because $\vec{F} \perp \vec{v}$

$B = mv/qr = \frac{(3 \cdot 10^{-26})(2 \cdot 10^6)}{(4 \cdot 10^{-20})(.25)} = 6$ Tesla

b) $T = \frac{2\pi r}{v} = \frac{2(3.14)(.25)}{2 \cdot 10^6} = .785 \cdot 10^{-6}$ sec

**3.** (40 points) In the circuit shown, the batteries have negligible internal resistance and the meters are both idealized. The emfs $E_1 = 30$ Volts and. $E_2 = 10$ Volts. The resistors have the values $R_1 = 80\Omega$ and $R_2 = 60\Omega$. (a) Find the reading of the voltmeter With the switch $S$ open. Which point is at higher potential: $a$ or $b$? (b) With the switch closed, find the reading of the voltmeter and the ammeter. Which way (up or down) does the current flow through the switch?




a) switch open the same current I runs around circuit

$+30 - I(80) + 10 - I(60) = 0$

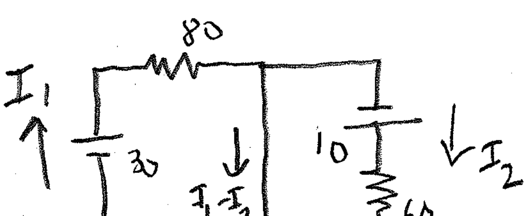
$E_1$ $R_1$ $E_2$ $R_2$

$I = 40/140 = .286$ A

Starting at "a" we gain 10 V going through $E_2$ and lose $IR_2 = 2/7 \; 60 = 120/7 = 17.1$ going through $R_2$ thus "a" is at higher potential (by 7.1 volts)

b) when S is closed $10 - 60 I_2 = 0 \Rightarrow I_2 = 1/6$ A

$30 - 80 I_1 = 0 \Rightarrow I_1 = 3/8$ A

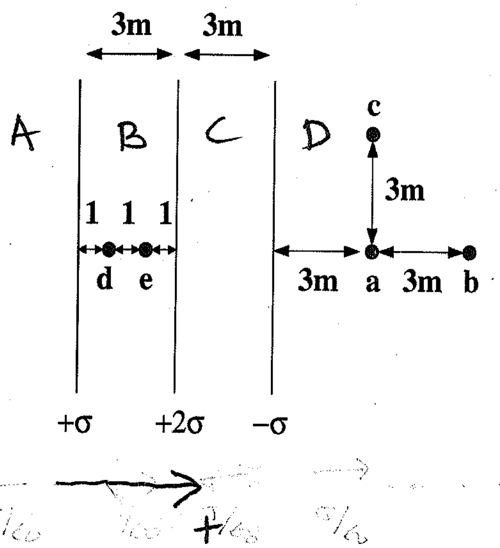



Through (A) flows $I_1 - I_2 = \frac{3}{8} - \frac{1}{6}$

$= \frac{9-4}{24} = 5/24$ Amp down

The voltmeter reads zero!

4. (30 points) Three infinite planes of charge have charge densities $+\sigma$ $+2\sigma$, and $-\sigma$ as shown.
Find the electric field everywhere. Give both magnitude and direction.
Find the potential difference $V_b - V_a$.
Find the potential difference $V_c - V_a$.
Find the potential difference $V_e - V_d$.




Electric field due to an infinite plane of charge is $\sigma/2\epsilon$, pointing away from plane if +, towards if −.

|  | E fields |  |  |  |
|---|---|---|---|---|
| Region | $+\sigma$ plane | $+2\sigma$ plane | $-\sigma$ plane | TOT |
| A | $-\sigma/2\epsilon_0$ | $-\sigma/\epsilon_0$ | $+\sigma/2\epsilon_0$ | $-\sigma/\epsilon_0$ |
| B | $+\sigma/2\epsilon_0$ | $-\sigma/\epsilon_0$ | $+\sigma/2\epsilon_0$ | 0 |
| C | $+\sigma/2\epsilon_0$ | $+\sigma/\epsilon_0$ | $+\sigma/2\epsilon_0$ | $2\sigma/\epsilon_0$ |
| D | $+\sigma/2\epsilon_0$ | $+\sigma/\epsilon_0$ | $-\sigma/\epsilon_0$ | $\sigma/\epsilon_0$ |

If E is uniform
$V = Ed\cos\theta$

$V_b - V_a = -\sigma/\epsilon_0 \; 3$ (b is at lower potential than a)

$V_c - V_a = 0$ ← Motion is ⊥ to $\vec{E}$ field

$V_e - V_d = 0$ ← $E = 0$ in region B.

5. (40 points) Charge $Q = +6.0\,\mu C$ is distributed uniformly over the volume of an insulating sphere that has radius $R = 3.0$ cm. What is the potential difference between the center of the sphere and the surface of the sphere? **Important:** A complete solution to this problem will provide the following steps:
(a) Write down Gauss' Law.
(b) Apply Gauss' Law to get the electric field inside the sphere ($r < R$). Explain clearly how you get the charge enclosed that you use in your equation.
(c) Write the equation which tells you how to get $V$ from $E$.

a) $\Phi_E = Q_{enclosed}/\epsilon_0$

b) $4\pi r^2 E = Q/\epsilon_0 \left[ \frac{4}{3}\pi r^3 / \frac{4}{3}\pi R^3 \right]$ ← ratio of volume enclosed to total volume since charge is uniform

$E = \frac{Q}{4\pi\epsilon_0} \, r/R^3 = kQr/R^3$

c) $V = -\int_R^0 E\,dr = -\int_R^0 kQr/R^3 = -kQr^2/2R^3 \Big|_R^0 = kQ/2R$

numerically $\frac{9\cdot 10^9 (6\cdot 10^{-6})}{2(.03)} = 9.0\cdot 10^5$ Volts

7. (10 points) How does the equivalent resistance of two resistors in parallel compare to the individual resistances $R_1$ and $R_2$? Why is this reasonable?

$1/R = 1/R_1 + 1/R_2 = R_1 + R_2 / R_1 R_2$

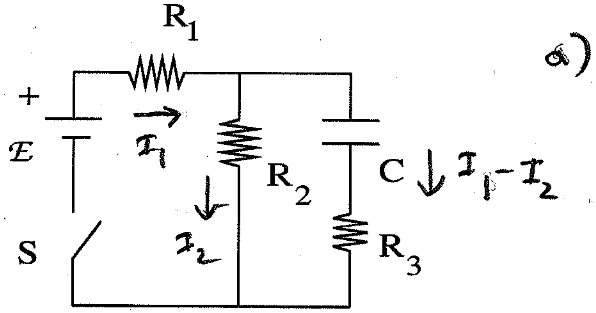
Equivalent resistance of resistors in parallel is less than individual resistors

$R = R_1 R_2 / R_1 + R_2$ clearly $R < R_1$ and $R < R_2$

because $R_1 R_2 / R_1 + R_2 < R_1 R_2 / R_1 = R_2$

Reasonable because current has 2 paths to choose from → lower resistance

[8.] (30 points) The capacitor $C = 2\mu F$ in the figure is initially uncharged. The resistances $R_1 = 10\Omega$, $R_2 = 5\Omega$, $R_3 = 20\Omega$, and the emf $E = 60$ volts. The switch is closed at $t = 0$. (a) Immediately after the switch is closed, what is the current through each resistor? (b) What is the current through each resistor after the capacitor is fully charged? (c) What is the final charge on the capacitor? (d) What is the final energy stored by the capacitor?




a) At $t=0$ voltage across C is zero since uncharged. Thus

$E - I_1 R_1 - (I_1 - I_2) R_3 = 0$

$E - I_1 R_1 - I_2 R_2 = 0$

$60 - 10 I_1 - (I_1 - I_2) 20 = 0$

$60 - 10 I_1 - I_2 5 = 0$

$60 - 30 I_1 + 20 I_2 = 0$

•4 $60 - 10 I_1 - 5 I_2 = 0$

$300 - 70 I_1 = 0$

$I_1 = 30/7$ A $\quad I_2 = 24/7$ A $\quad I_3 = I_1 - I_2 = 6/7$ A

b) After full charging, no current flows into capacitor and hence $I_3 = I_1 - I_2 = 0 \quad I_1 = I_2$

$60 - I_1 10 - I_1 5 = 0 \quad I_1 = 4$ A

$= I_2$

[9.] (10 points) What is the reason one puts a dielectric between the plates of a capacitor?

The dielectric reduces the electric field between the plates and hence the voltage between the plates. One is able to store more charge at the same voltage.