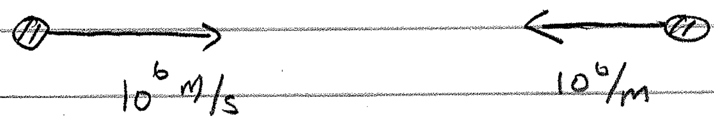
1.

Physics 9C
Homework 3
Winter 2016

ch 23	12, 20, 42, 46, 47, 73	
ch 24	3, 6, 11, 22, 36, 53, 60	
EEC 7-3		

$\boxed{23-12}$



Energy conservation

$$2\left\{\frac{1}{2}(1.67\cdot 10^{-27})(10^6)^2\right\} + \phi = \phi + \frac{k q_1 q_2}{r}$$

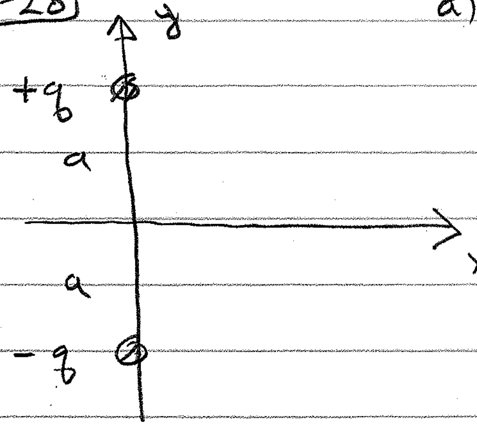
KE PE KE

$$1.67\cdot 10^{-15} = 9\cdot 10^9 (1.6\cdot 10^{-19})^2 / r$$

$$r = 1.38\cdot 10^{-14}\ m$$

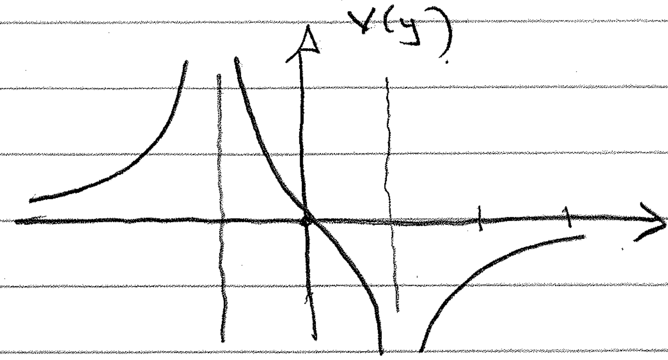
$$F = k q_1 q_2 / r^2 = 9\cdot 10^9 \frac{(1.6\cdot 10^{-19})^2}{(1.38\cdot 10^{-14})^2} = 1.21\cdot 10^{-1}\ N$$

$\boxed{23-20}$



a) $V = \dfrac{-kq}{|y-a|} + \dfrac{kq}{|y+a|}$

b)



c) If $y > a$ $\quad |y-a| = y-a$

$|y+a| = y+a$

$$V = \frac{-kq}{y-a} + \frac{kq}{y+a} = kq\left\{\frac{-y-a+y-a}{y^2-a^2}\right\}$$

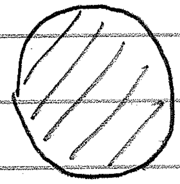
$$= -2kqa/y^2-a^2$$

d) $kq/|y-a| - kq/|y+a|$

and $+2kqa/y^2-a^2$

3.

$\boxed{23-42}$ a) $V = kQ/R = \frac{9 \cdot 10^9 Q}{.25} = 1500$



$R = .25$

Q

$Q = 6.67 \cdot 10^{-7}$ C

Note $V_{center} = V_{surface}$ since $E=0$ inside conductor

b) $V = 1500$ V at surface also (see a)

$\boxed{23-46}$ $V = Ax^2y - Bxy^2$ $\quad A = 5\ V/m^3$, $\quad B = 8\ V/m^3$

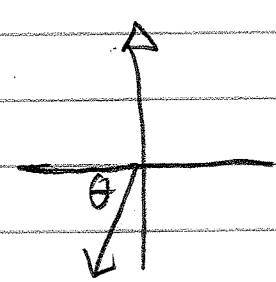
$$\vec{E} = -\nabla V = -\partial/\partial x\, V\ \hat{x} - \frac{\partial V}{\partial y}\hat{y}$$

$$= \left(-2Axy + By^2\right)\hat{x} + \left(-Ax^2 + 2Bxy\right)\hat{y}$$

$$\vec{E} = \left(-2(5)(2)(.4) + 8(.4)^2\right)\hat{x} + \left(-5(2)^2 + 2(8)(2)(.4)\right)\hat{y} = -6.72\hat{x} - 13.6\hat{y}$$

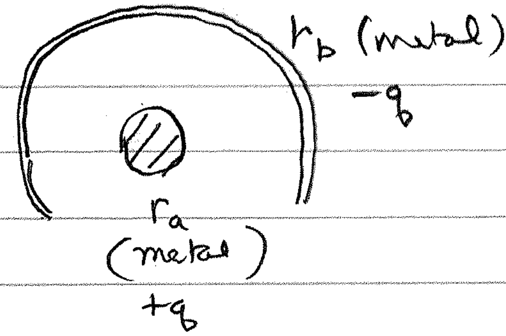
$|\vec{E}| = 15.2$ N/C

direction:



$\theta = \tan^{-1}\frac{13.6}{6.72} = 63.7^\circ$

$\boxed{23-47}$



a) $\vec{E} = 0$ outside so $V = 0$ for $r > r_b$

$E = kq/r$ for $r_a < r < r_b$

so $V = -\int_{r_b}^{r} \frac{kq}{r'^2}\,dr' = \frac{kq}{r} - \frac{kq}{r_b}$

$\vec{E} = 0$ inside $\overset{\text{central}}{\wedge}$ metal sphere so $V = \frac{kq}{r_a} - \frac{kq}{r_b}$ for $r < r_a$

b) $V(r_a) - V(r_b) = kq\left(1/r_a - 1/r_b\right)$ from part a

c) $\vec{E} = -\underset{d/dr}{\vec{\nabla}}V = +kq/r^2\ \hat{r}$

From (b) $kq = V_{ab}\left(1/r_a - 1/r_b\right)^{-1}$

so $E = \frac{V_{ab}}{1/r_a - 1/r_b}\ \frac{1}{r^2}\ \hat{r}$

d) $\vec{E} = -\vec{\nabla}V = 0$

5.

$\boxed{23-47}$ cont'd

e) Redoing problem

$r > r_b \qquad E = \frac{k(q-Q)}{r^2}$

$r_a < r < r_b \qquad E = kq/r^2$

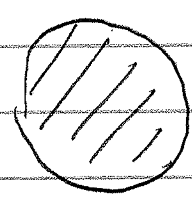
$r < r_a \qquad E = 0$

$V(r_b) = k(q-Q)/r_b^2$

$V(r_a) - V(r_b) = -\int_{r_b}^{r_a} kq/r^2 \, dr = kq/r \Big|_{r_b}^{r_a}$

$= kq\left(1/r_a - 1/r_b\right)$

$\boxed{23-73}$



Inside sphere

$(4\pi r^2)E = \frac{1}{\epsilon_0} \frac{\frac{4}{3}\pi r^3 Q}{\frac{4}{3}\pi R^3}$

$Q = 4\mu C$

$R = .05 \text{ cm} \qquad E = \frac{1}{4\pi\epsilon_0} Q r/R^3$

Integrating to get $V_{center} - V_{surface}$

$V(0) - V(R) = -\int_R^0 \frac{kQr}{R^3} dr = -\frac{kQr^2}{2R^3}\Big|_R^0 = \frac{kQ}{2R}$

6.

$\boxed{24\text{-}3}$ (a) $C = Q/V$

$V = Q/C = 1.48 \cdot 10^{-7}\ C / 2.45 \cdot 10^{-10}\ F = 604$ Volts

(b) $C = \epsilon_0 A/d = (8.85 \cdot 10^{-12}) A / 3.28 \cdot 10^{-4} = 2.45 \cdot 10^{-10}$

$A = 9.1 \cdot 10^{-3}\ m^2$

(c) $E = V/d = 604 / 3.28 \cdot 10^{-4} = 1.84 \cdot 10^{6}\ N/C$

(d) $E = \sigma/\epsilon_0 \quad \sigma = \epsilon_0 E = (8.85 \cdot 10^{-12})(1.84 \cdot 10^{6})$

$= 1.63 \cdot 10^{-5}\ C/m^2$

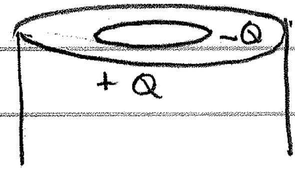
check $\sigma A = (1.63 \cdot 10^{-5})(9.1 \cdot 10^{-3}) = 1.48 \cdot 10^{-7}\ C$ ✓

$\boxed{24\text{-}6}$ (a) 12 Volts

(b) $E = \sigma/\epsilon_0$ is the same regardless of d
so if d is doubled $V = Ed$ is doubled
$V = 24$ volts

(c) If r is doubled $A \to 4A$
so $\sigma \to \sigma/4$, Thus $E \to E/4$
and $V \to V/4$ $V = 3$ Volts.

$\boxed{24-11}$



$Q = 10\,pC$

$r_2 = 5 \cdot 10^{-3}\,m$

$r_1 = 5 \cdot 10^{-4}\,m$

$l = 0.18\,m$

$$\overbrace{E(2\pi r l)}^{\Phi_E} = Q/\epsilon_0$$

$$E = Q/2\pi\epsilon_0 r l$$

$$V = \int_{r_1}^{r_2} E\,dr = \frac{Q}{2\pi\epsilon_0 l} \ln r_2/r_1$$

(a) $$C = \frac{Q}{V} = 2\pi\epsilon_0 l/\ln(r_2/r_1)$$

$$= 2\pi(8.85 \cdot 10^{-12})(.18)/\ln 10 = 4.34 \cdot 10^{-12}\ F$$

(b) $$V = Q/C = 10^{-11}/4.34 \cdot 10^{-12} = 2.3\ \text{Volts}$$

$\boxed{24-22}$ (a) $5 + 8\,\mu F$ in parallel $C = 13\,\mu F$

The rest in series $\frac{1}{C} = \frac{1}{10} + \frac{1}{13} + \frac{1}{9}$

$$C = 3.47\,\mu F$$

8.

24-22 cont'd $\qquad C = Q/V$

(b) $Q = CV = (3.47 \cdot 10^{-6})(50) = 1.74 \cdot 10^{-4}\ C$

(c) $\cdot$ Q on 10, 9 μF capacitors is $1.7\cancel{4} \cdot 10^{-4}\ C$

<u>check</u> potentials across 10, 9 μF capacitors are

$$V = Q/C = \frac{1.7\cancel{4} \cdot 10^{-4}}{10 \cdot 10^{-6}} = 17.4\ V$$

$$\frac{1.7\cancel{4} \cdot 10^{-4}}{9 \cdot 10^{-6}} = 19.3\ V$$

potential across 5 and 8 μF capacitors is

$$50 - 17.4 - 19.3 = 13.3 \text{ Volts}$$

charge on $5\mu F$: $CV = 6.65 \cdot 10^{-5}$
$8\mu F$: $CV = 10.64 \cdot 10^{-5}$

adding gives $17.3 \cdot 10^{-5} C$ ✓✓

9.

$\boxed{24-36}$ (a) $E = \sigma/\epsilon_0$ $\quad C = \epsilon_0 A/d$

$= Q/A\epsilon_0$

$E = Q/Cd$

If $E < 3 \cdot 10^4$ must have

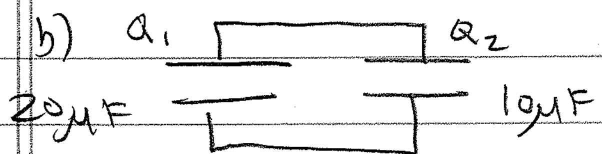
$Q < (3 \cdot 10^4)(5 \cdot 10^{-12})(1.5 \cdot 10^{-3}) = 2.25 \cdot 10^{-10}$ C

(b) Dielectric reduces E allowing Q to be factor of 2.7 larger

$\boxed{24-53}$ $C = Q/V$

a) $Q = CV = (20 \cdot 10^{-6})(800) = 1.6 \cdot 10^{-2}$ C

b)



$\frac{Q_1}{20 \cdot 10^{-6}} = \frac{Q_2}{10 \cdot 10^{-6}}$

so $Q_1 = 2Q_2$

also $Q_1 + Q_2 = 1.6 \cdot 10^{-2}$

so $Q_2 = 0.53 \cdot 10^{-2}$ C

$Q_1 = 1.06 \cdot 10^{-2}$ C

$V_1 = V_2 = 530$

24-53 cont'd

c) $U_1 = Q_1^2/2C_1 = (1.06 \cdot 10^{-2})^2/2(20 \cdot 10^{-6})$

$= 2.81\ J$

$U_2 = Q_2^2/2C_2 = (0.53 \cdot 10^{-2})^2/2(10 \cdot 10^{-6})$

$= 1.40\ J$

$$U_0 = \frac{Q^2}{2C} = \frac{(1.6 \cdot 10^{-2})^2}{2(20 \cdot 10^{-6})} = 6.40\ J$$

general formulae $Q_1 + Q_2 = Q_0$

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad Q_2 = Q_1 C_2/C_1$

$\rightarrow \quad Q_1(1 + C_2/C_1) = Q_0 \qquad Q_1 = Q_0 C_1/C_1 + C_2$

$Q_2 = Q_0 C_2/C_1 + C_2$

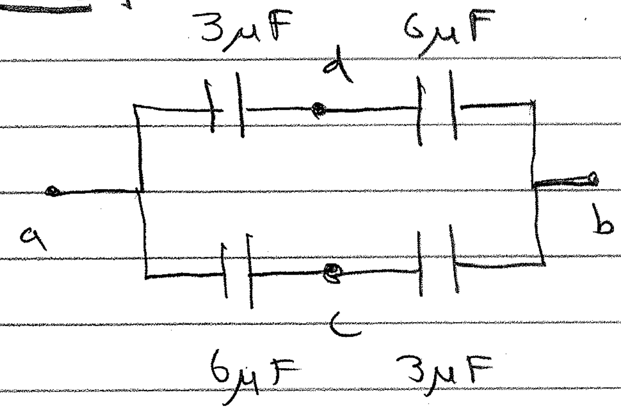
$$U_{final} = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{Q_0^2}{2(C_1+C_2)^2}(C_1 + C_2)$$

$= Q_0^2/2(C_1 + C_2)$

$\Rightarrow \quad U_{final} \leq U_{initial}$ always! (as expected)

11.

$\boxed{24-60}$



$V_{ab} = 210$

All capacitors have same charge Q with switch open

This is because charges on 2 top capacitors must be equal and because top + bottom loops identical

$C = Q/V$

$V = Q/C$

$210 = Q/3 \cdot 10^{-6} + Q/6 \cdot 10^{-6}$

$= Q/10^{-6} \{ 1/3 + 1/6 \}_{= 1/2}$

$Q = 4.2 \cdot 10^{-4}$ C

V across 3μF is $\frac{4.2 \cdot 10^{-4}}{3 \cdot 10^{-6}} = 140$ V

6μF is 70 V

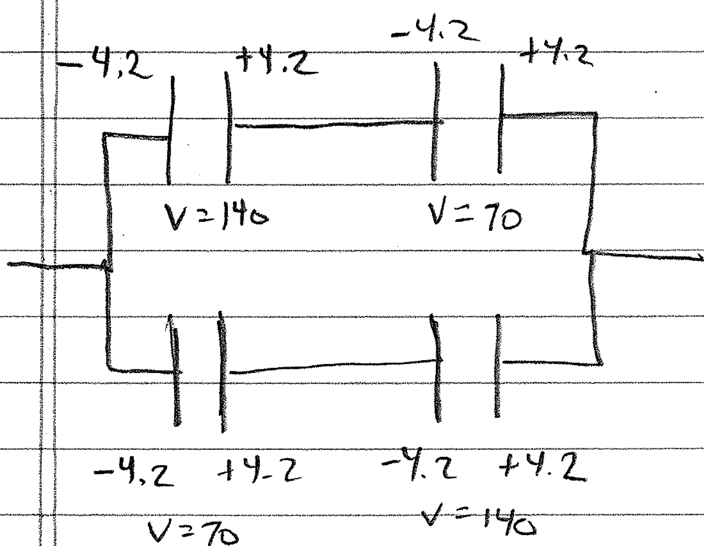
so $V_{cd} = 70$ V

24-60 cont'd

When switch closed potential across $3\mu F$ and $6\mu F$ must equal. By symmetry, potential must be 105 V (1/2 of 210)

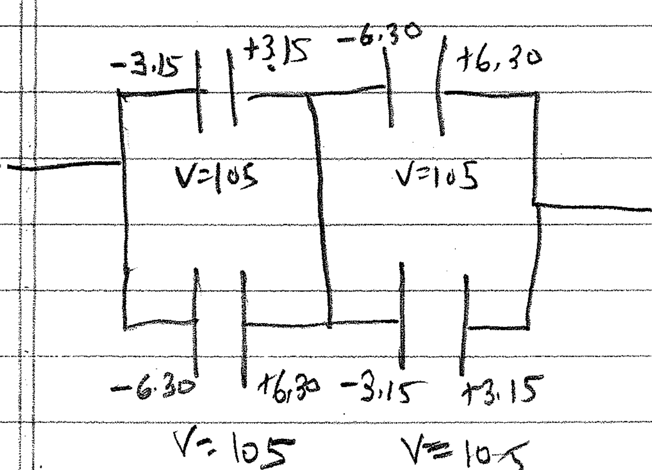
$$\frac{Q_{on 3}}{3 \cdot 10^{-6}} = 105 \qquad Q_{on 3} = 3.15 \cdot 10^{-4}\, C$$

$$Q_{on 6} = 6.30 \cdot 10^{-4}\, C$$



← Switch open

Charge through switch was $3.15 \cdot 10^{-4}$ C.



← switch closed

EEC 130A 7-3

Between 2 plates separated by vacuum $E = 4\pi k\sigma = \sigma/\epsilon_0$
If dielectric is present E is reduced $E = \sigma/\epsilon_1$ or σ/ϵ_2

The electric fields in sections 1, 2 must be the same, because the potential across the plates is the same whether one takes path through dielectric 1 or dielectric 2.

So charge densities σ_1 and σ_2 must be different such that $\sigma_1/\epsilon_1 = \sigma_2/\epsilon_2$. $V = \sigma_1/\epsilon_1\, d = \sigma_2/\epsilon_2\, d$

By definition $C = Q/V = \dfrac{\sigma_1 A_1 + \sigma_2 A_2}{\frac{\sigma_1}{\epsilon_1} d}$

$$= \frac{\epsilon_1 A_1}{d} + \frac{\sigma_1 \epsilon_2/\epsilon_1\, A_2}{\sigma_1/\epsilon_1\, d} = \underbrace{\frac{\epsilon_1 A_1}{d}}_{C_1} + \underbrace{\frac{\epsilon_2 A_2}{d}}_{C_2}$$

Can also do this by analysis of energy stored

$$\frac{1}{2} C V^2 = \frac{1}{2}\epsilon_1 E_1^2 (dA_1) + \frac{1}{2}\epsilon_2 E^2 (dA_2)$$

$$\frac{1}{2} C\left(\frac{\sigma_1}{\epsilon_1} d\right)^2 = \frac{1}{2}\epsilon_1\left(\frac{\sigma_1}{\epsilon_1}\right)^2 dA_1 + \frac{1}{2}\epsilon_2\left(\frac{\sigma_1}{\epsilon_1}\right)^2 dA_2$$

$d\,\frac{1}{2}(\sigma/\epsilon_1)^2$ cancel everywhere

$$C = \epsilon_1 A_1/d + \epsilon_2 A_2/d$$