1.

Physics 9C W 2016
Assignment 2

$\boxed{22-Q8}$ Gauss' law would fail. The flux through a spherical surface of radius R would be

$$\Phi_E = \int \vec{E}\cdot\hat{n}\,dA = \frac{kQ}{R^3}4\pi R^2 = kQ/R$$

This depends on R, not just on the charge Q.

$\boxed{22-6}$ $\vec{E} = (2.4\cdot10^4\,\hat{i} + 3.2\cdot10^4\,\hat{j})\,N/C$

since $\cos 53.1 = 0.6$ and $\sin 53.1 = 0.8$

Each side of cube has area $(0.1\,m)^2 = 0.01\,m^2$

Side	(outward) normal vector $\hat{n}$	$\vec{E}\cdot\hat{n}$	$\Phi_E = \vec{E}\cdot\hat{n}\,dA$
S_1	$-\hat{j}$	$-3.2\cdot10^4$ N/C	$-3.2\cdot10^2$ Nm²/C
S_2	$+\hat{k}$	0 "	0 "
S_3	$+\hat{j}$	$+3.2\cdot10^4$ "	$+3.2\cdot10^2$ "
S_4	$-\hat{k}$	0 "	0 "
S_5	$+\hat{i}$	$+2.4\cdot10^4$ "	$+2.4\cdot10^2$ "
S_6	$-\hat{i}$	$-2.4\cdot10^4$ "	$-2.4\cdot10^2$ "

$\Phi_E^{TOT} = \phi$ There is no net charge inside cube.

2.

$\boxed{22-14}$ For a metal object, all charge flows to surface and there is no $\vec{E}$ field inside so (b) $\vec{E} = 0$.
Outside the sphere, we use Gauss' law (or Coulomb's law)

$$(4\pi R^2)E = 4\pi k Q \qquad E = kQ/R^2$$

$$E = 9 \cdot 10^9 \, (.25 \cdot 10^{-9}) / (.55)^2 = 7.44 \text{ N/C}$$

$\boxed{22-23}$ From Gauss' law $4\pi R^2 E = 4\pi k Q$

Thus $Q = ER^2/k = 1750\,(.5)^2/9 \cdot 10^9 = 4.86 \cdot 10^{-8}$ C

$.355 + .145$

The charge density $\rho = Q/\tfrac{4}{3}\pi R^3 = \dfrac{4.86 \cdot 10^{-8}}{\tfrac{4}{3}\pi(.355)^3} = 2.59 \cdot 10^{-7} \, \dfrac{C}{m^3}$

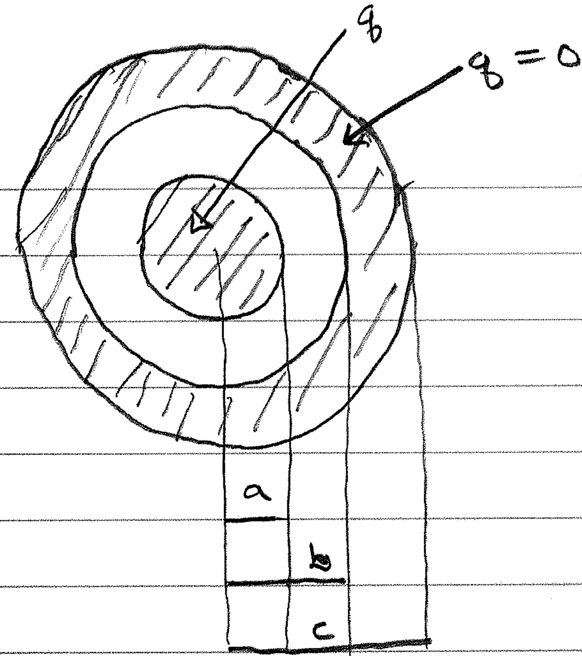
By Gauss' law $4\pi r^2 E = 4\pi k Q_{enc} = 4\pi k \rho \tfrac{4}{3}\pi r^3$

thus $E = k\rho \tfrac{4}{3}\pi r$

$$= 9 \cdot 10^9 (2.59 \cdot 10^{-7}) \tfrac{4}{3}\pi (.2) = 1.95 \cdot 10^3 \text{ N/C}$$

3.

$\boxed{22-44}$

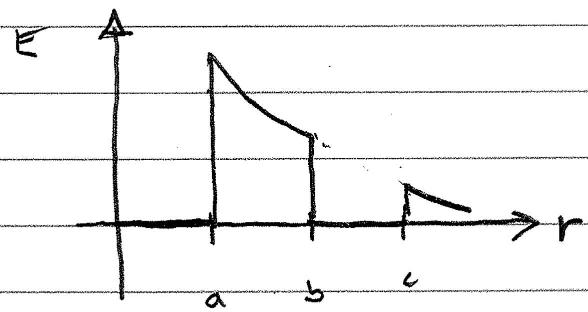


$\vec{E} = 0$ inside metal object

$\Rightarrow$ $\vec{E} = 0$ for $r < a$ and for $b < r < c$

For $a < r < b$ $\quad 4\pi r^2 E = 4\pi k q$, so $\quad E = kq/r^2$

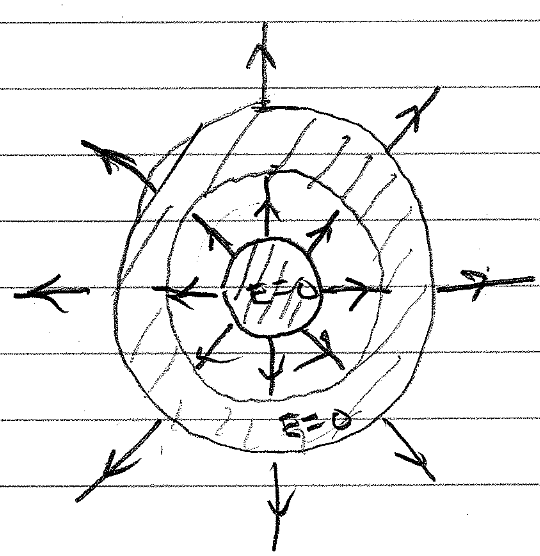
Similarly for $r > c$ $\quad E = kq/r^2$



c) charge on inner surface of hollow sphere $= -q$ so that $E = 0$ for $b < r < c$

d) Since total q on hollow sphere $= 0$, charge on outer surface $= +q$

e)



4.

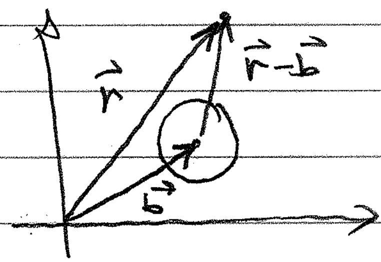
$\boxed{22-61}$ We know that a distance r from the center of a sphere of charge density ρ (inside the sphere)

$$|\vec{E}|4\pi r^2 = 4\pi k\,\rho\,\tfrac{4}{3}\pi r^3 \rightarrow |\vec{E}| = k\rho\frac{4\pi r}{3}$$

or, since $k = \frac{1}{4\pi\epsilon_0}$ $\qquad |\vec{E}| = \rho r/3\epsilon_0$

If sphere is at origin $\vec{E} = \rho r/3\epsilon_0\,\hat{r} = \rho\vec{r}/3\epsilon_0$

For a sphere whose center is at $\vec{b}$, the position vector to the center is $\vec{r}-\vec{b}$: see figure



thus $\vec{E} = \rho\frac{\vec{r}-\vec{b}}{3\epsilon_0}$

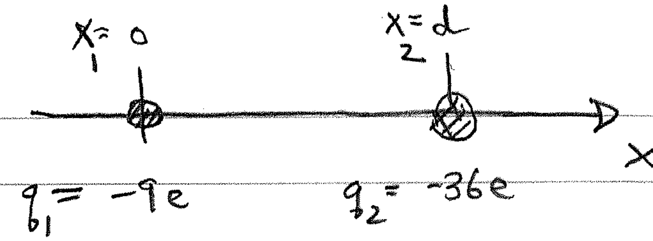
The electric field inside a complete sphere would be

$\vec{E}_{complete} = \rho\vec{r}/3\epsilon_0$. To get $\vec{E}$ inside the hole, we need to subtract the part of $\vec{E}_{complete}$ due to the missing material

$$\vec{E} = \rho\vec{r}/3\epsilon_0 - \rho(\vec{r}-\vec{b}/3\epsilon_0) = \rho\vec{b}/3\epsilon_0$$

5.

$\boxed{\text{EEC 130A 5-5}}$



$$\left.\begin{aligned}
\vec{F}_1 &= \text{Force on 1} = \vec{F}_{2\,on\,1} + \vec{F}_{3\,on\,1} \\
\vec{F}_2 &= \text{Force on 2} = \vec{F}_{1\,on\,2} + \vec{F}_{3\,on\,2} \\
\vec{F}_3 &= \text{Force on 3} = \vec{F}_{1\,on\,3} + \vec{F}_{2\,on\,3}
\end{aligned}\right\}$$

Clearly $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ From Newton's 3rd Law $\vec{F}_{i\,on\,j} = -\vec{F}_{j\,on\,i}$

Problem says "equilibrium" means $\vec{F}_1 = \vec{F}_2 = \vec{F}_3$

If $\vec{F}_1 = \vec{F}_2 = \vec{F}_3$ and $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ we must

have $\vec{F}_1 = \vec{F}_2 = \vec{F}_3$ which is the more natural definition of equilibrium.

If we put a positive charge q_3 at $d/3$ it

will have $\vec{F}_{1\,on\,3} = \dfrac{-k(+9e)q_3}{(d/3)^2}\,\hat{i} = \dfrac{-81keq_3}{d^2}\,\hat{i}$

$$\vec{F}_{2\,on\,3} = \frac{+k(+36e)q_3}{(2d/3)^2} = +\frac{81keq_3}{d^2}\,\hat{i}$$

So $\vec{F}_3 = 0$ for any magnitude or size of q_3!

6.

cont'd Check other charges. For q_2

$$\vec{F}_{1 \text{ on } 2} = k\frac{(-9e)(-36e)}{d^2}\hat{\imath} = 324\,ke^2/d^2\,\hat{\imath}$$

$$\vec{F}_{3 \text{ on } 2} = k\frac{q_3(-36e)}{(2d/3)^2}\hat{\imath} = -81\,kq_3e/d^2$$

For these to balance out we need $q_3 > 0$ and

also $81 q_3 e = 324\, e^2 \implies q_3 = {}^{+}4e$

We can verify q_1 is also in equilibrium

$$\vec{F}_{3 \text{ on } 1} = \frac{k(9e)(4e)}{(d/3)^2}\hat{\imath} = k\,324e^2/d^2\,\hat{\imath}$$

$$\vec{F}_{2 \text{ on } 1} = -\frac{k(9e)(36e)}{d^2}\hat{\imath} = -k\,324e^2/d^2\,\hat{\imath} \quad \checkmark$$

$\boxed{\text{EEC 130A 6-1}}$ We have $\oint \vec{E}\cdot\hat{n}\,dA = Q_{enc}/\epsilon_0$

(a) and $Q_{enc} = \int dV\,\rho(r)$

By Gauss' law $\oint \vec{E}\cdot\hat{n}\,dA = \oint \vec{\nabla}\cdot\vec{E}\,dV$

So $\int \vec{\nabla}\cdot\vec{E}\,dv = \int \rho/\epsilon_0\,dV$. Since this is true for all volumes we must have $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

EEC 130A 6-1 cont'd.

The vector $\vec{D}$ is related to $\vec{E}$ by $\vec{E} = 4\pi k \vec{D}$.

That is, $\oint \hat{D} \cdot \hat{n}\, dA = Q_{enc}$

the flux of $\hat{D}$ through a surface just measure the charge enclosed with no $4\pi k$ factor (no $1/\epsilon_0$ factor)

So $\vec{\nabla} \cdot \vec{D} = \rho$

Here $\vec{D} = \hat{x}\, 2(x+y) + \hat{y}\,(3x-y)$

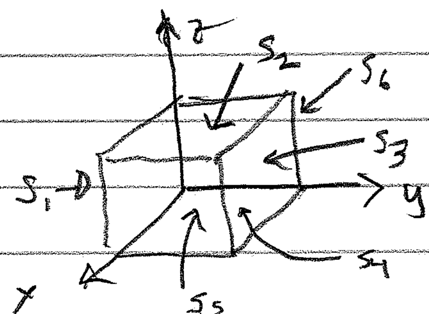
So $\vec{\nabla} \cdot \vec{D} = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

$= 2 - 1 = \underline{1}$

There must be a uniform charge density $\rho_v = 1\ C/m^3$ in all of space

(b) $Q = \int \rho\, dV = \rho \int dV = \rho V$ (if ρ = const) (V: $8\ m^3$) $\quad Q = 8C$.

c) This is very similar to 22-6!



$\hat{\imath}$ same $\hat{\jmath}$

$\vec{D} = \hat{x}(2)(x+y) + \hat{y}(3x-y)$

EEC 130A 6-1 cont'd

side	$\hat{n}$	description	$\vec{D}\cdot\hat{n}$
S_1	$-\hat{j}$	$y=0 \quad 0 \le x, z \le 2$	$-3x+y$
S_2	$+\hat{k}$	$z=2 \quad 0 \le x, y \le 2$	0
S_3	$+\hat{j}$	$y=2 \quad 0 \le x, z \le 2$	$3x-y$
S_4	$-\hat{k}$	$z=0 \quad 0 \le x, y \le 2$	0
S_5	$+\hat{\imath}$	$x=2 \quad 0 \le y, z \le 2$	$2(x+y)$
S_6	$-\hat{\imath}$	$x=0 \quad 0 \le y, z \le 2$	$-2(x+y)$

Flux of $\vec{D}$ through S_1 is

z integral x integral

$$\int_0^2 dx \int_0^2 dz\,(-3x) = 2\left(-\frac{3x^2}{2}\right)\Big|_0^2 = -12$$

S_2 and S_4 give ϕ flux

S_3 $\quad \int_0^2 dx \int_0^2 dz\,(3x-2) = 2\ \frac{3x^2}{2} - 2x\Big|_0^2 = +4$

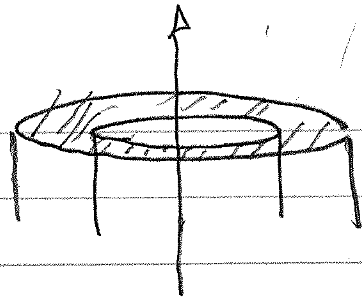
S_5 $\quad \int_0^2 dy \int_0^2 dz\,(4+2y) = 2\ 4y + y^2\Big|_0^2 = 24$

S_6 $\quad \int_0^2 dy \int_0^2 dz\ -2y = 2\ -y^2\Big|_0^2 = -8$

adding up $\rightarrow$ 8 !!

9.

$\boxed{\text{EEC 130A 6-2}}$



$r_{inside} = 1$

$r_{outside} = 3$

charge density ρ is uniform

$0 < r < 1 \rightarrow$ no charge enclosed $\rightarrow D = 0$

$1 < r < 3$ charge enclosed $= (\pi r^2 - \pi 1^2) L \rho$

$$\underbrace{L\, 2\pi r\, |\vec{D}|}_{\text{flux of } \vec{D}} = \text{charge enclosed}$$

$$|\vec{D}| = \frac{(r^2-1)\rho}{2r}$$

$3 < r$ $\quad L\, 2\pi r |\vec{D}| = (\pi 3^2 - \pi 1^2) L \rho$

$$|\vec{D}| = 4\rho / r$$