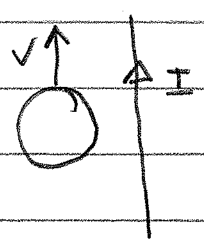
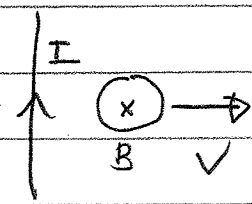
1

Physics 9C W2016
Homework 6 Solution

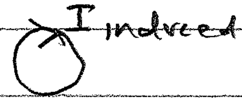
1 The induced EMF is given by $d\Phi_B/dt$, the time rate of change of the magnetic flux. This is Faraday's law. The direction of the induced current, by Lenz' law, must <u>oppose</u> the change in Φ_B.

The field due to a current carrying wire is $B = \mu_0 I / 2\pi r$. In this geometry, the flux Φ_B is unchanging because neither I changes nor the distances r to the wire. On the other hand, here





the distances r are increasing, so the field B is decreasing. By the right hand rule B is into the paper, so as the loop moves the flux Φ_B into the paper <u>decreases</u>. Lenz' law tells us the current induced must fight this decrease, ie must <u>increase</u> the flux into the paper. Thus I induced must be clockwise



← Right hand rule ⇒ Φ_B from induced current is into paper.

2.

2

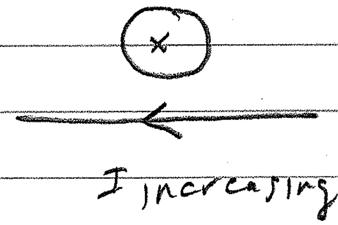
$$\Phi_B = \overbrace{\pi(.13)^2}^{A}\overset{B}{(.9)} \quad \text{initially}$$

$$\Phi_B = 0 \quad \text{finally} \Rightarrow \Delta\Phi_B = \pi(.13)^2(.9)$$

$\Delta t = .15$ sec.

Thus $\Delta\Phi_B/\Delta t = \pi(.13)^2(.9)/.15 = .319$ Volts

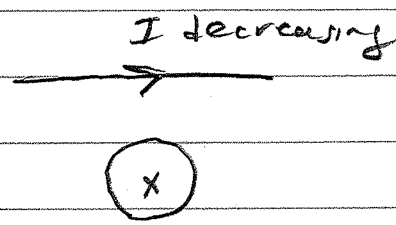
3 a)



Φ_B into paper is increasing since $B = \mu_0/2\pi r \; I$ and I is increasing

Induced current must oppose by creating Φ_B out of paper. $\Rightarrow$ I is counterclockwise

b)

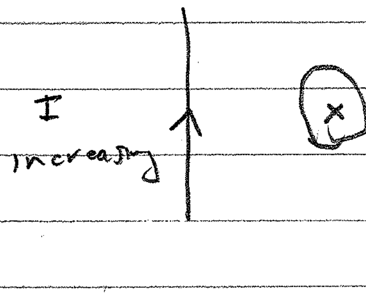


Φ_B into paper decreasing

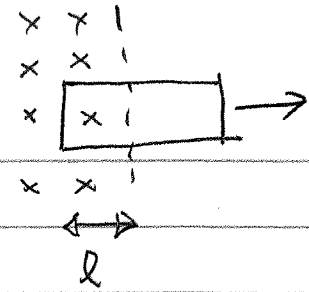
$\therefore$ Induced current needs to increase Φ_B into paper $\Rightarrow$ clockwise

c) $d\Phi_B/dt = 0 \Rightarrow$ no induced current

d)



Same as (a)



4

$\Phi_B = BA \qquad A = .35 \cdot l$

$d\Phi_B/dt = B\, dA/dt = 0.45 \; .35 \frac{dl}{dt}$

$\uparrow v = 3.4$

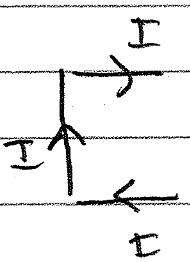
$d\Phi_B/dt = (.45)(.35)(3.4) = .536$

$I = .536/.23 = 2.33\ A$

$\uparrow R$

Force on vertical piece of wire is $I l B$

$= 2.33(.35)(.45) = .367\ N$



Note 1: Since Φ_B into paper is decreasing, I induced must increase Φ_B into paper.
so current runs upward through vertical part of wire
The force from B on the wire is to the left
from $I \vec{l} \times \vec{B}$.

Note 2: Power provided by pulling force is Fv

$= (.367)(3.4) = 1.25$ watts

Energy dissipated is $I^2 R = (2.33)^2 (.23) = 1.25$ watts

Energy is conserved

4.

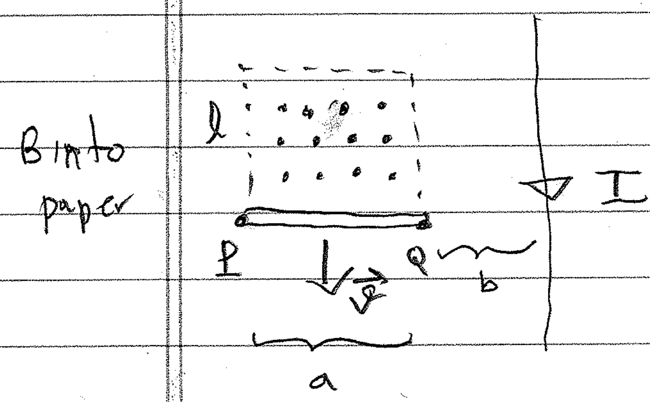
5 $\phi_B = BA$

$d\phi_B/dt = B\, dA/dt$

$= (.48)(.035) = .0168$ Volts

This answer is correct at $t=0$ and $t=2$ because dA/dt is constant. The initial area of the loop is irrelevant! (As long as loop doesn't shrink to nothing...)

6 We can make an "imaginary" current loop and compute the rate of change of flux.



$$\phi_B = l \int_b^{b+a} \frac{\mu_0 I}{2\pi r}\, dr$$

$$= l \frac{\mu_0 I}{2\pi} \ln r \Big|_b^{b+a}$$

$$= \frac{\mu_0 I l}{2\pi} \ln\left(\frac{b+a}{b}\right)$$

$$\frac{d\phi_B}{dt} = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{b+a}{b}\right) \quad \text{since } \frac{dl}{dt} = v$$

If v is in the same direction as I, ϕ_B into the paper is increasing. By Lenz' law the induced current must decrease ϕ_B into paper, ie be counterclockwise. Charge would flow from P to Q so P is at higher potential. The answer is reversed if $\vec{v}$ is reversed

You can also do this problem by using $\vec{F} = q\vec{v} \times \vec{B}$

$$|F| = q v \frac{\mu_0 I}{2\pi r}$$

Integrating F gives the work done in moving a charge from Q to P. This work is stored as potential energy

7. $\Phi_B = NAB \sin \omega t$

$d\Phi_B/dt = \underbrace{NAB\omega}\cos\omega t$ ↑

peak value since cosine is between -1 and 1

$$24 = N(.07)^2 \underbrace{2\pi 60}_{\omega}$$

8 Before motor starts turning $\omega = 0$ and hence $d\Phi_B/dt = NAB\omega\cos\omega t = 0$. There is no "back EMF" in the motor to reduce the voltage from the usual 120 Volt wall socket and so the current drawn is large. As motor speeds up and ω becomes large $d\Phi_B/dt$ is big and the voltage $V - d\Phi_B/dt$ ← "back EMF" is reduced.

8 (cont'd)

The motor draws less current.

A heater has no moving parts. It uses V/R immediately and always

9.

$$\phi_B = NAB \qquad B = B_0 e^{-t/\tau} \quad (B_0 = .5,\ \tau = .1)$$

$$= 10(\pi .1^2)(.5)e^{-10t}$$

$$V = \left| d\phi_B/dt \right| = 10(\pi(.1)^2)(.5)\,10e^{-10t}$$

$$= \frac{\pi}{2} e^{-10t}$$

$$\text{Power} = V^2/R = \frac{\pi^2}{8} e^{-20t}$$

($R = 2$)

10.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$|\vec{\tau}| = NIAB = 300(I)(.1)(.15)(.6)$$

(N = 300, A = (.1)(.15), B = .6)

$$FR = 250(.25) \qquad I = \frac{250(.25)}{300(.1)(.15)(.6)} = 23\ A$$

$$\text{Back EMF} = NAB\omega = 300(.1)(.15)(.6)(33.3)$$

$$= 90.\ \text{Volts}$$

$$\omega = V/R = \frac{30 \cdot 10^3}{3600\ (.25)} = 33.3$$

7.

10 12 volt batteries
BACK EMF

c) Power in coils $(120-90)\,23 = 690$ Watts

net V — I

d) Power to push car $= Fv$

$= 250\,\frac{30 \cdot 1000}{3600} = 2083$ Watts

$\frac{2083}{2083+690} = .75$