1.

Physics 9C W2016

Homework 5

$\boxed{1}$ The field due to a solenoids is $B = \mu_0 n I$ where n = # turns per length. Let's suppose our wire has cross section a. If we have a given mass m, the length ℓ of wire is determined by $M = \rho a \ell$ where ρ = density. If the solenoid has radius R then we can go around it $N = \ell/2\pi R$ times. If its length is L then $n = N/L = \ell/2\pi R L$

We conclude $B = \mu_0 I n = \mu_0 I \frac{\ell}{2\pi R L} = \mu_0 I \frac{M}{\rho a 2\pi R L}$

The quantities $\mu_0, I, M, \rho, 2\pi$, are given to us.

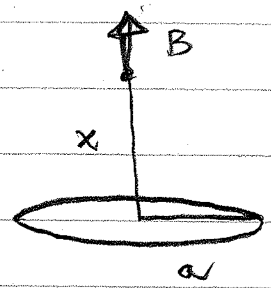
We want to make a, R, L small to maximize B.

This makes sense: if the cross section a of the wire is small we can have a lot of wire to wrap around. Likewise if the radius R of the solenoid is small we can go around it many times. Making L small is a bit uncertain: the formula $B = \mu_0 n I$ really applies only to a long solenoid, so making L small will destroy that formula.

2.

2. The field due to a current loop is (along its axis)

$$B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}} \qquad a = \text{radius.}$$



In this problem $a = R_e = x$

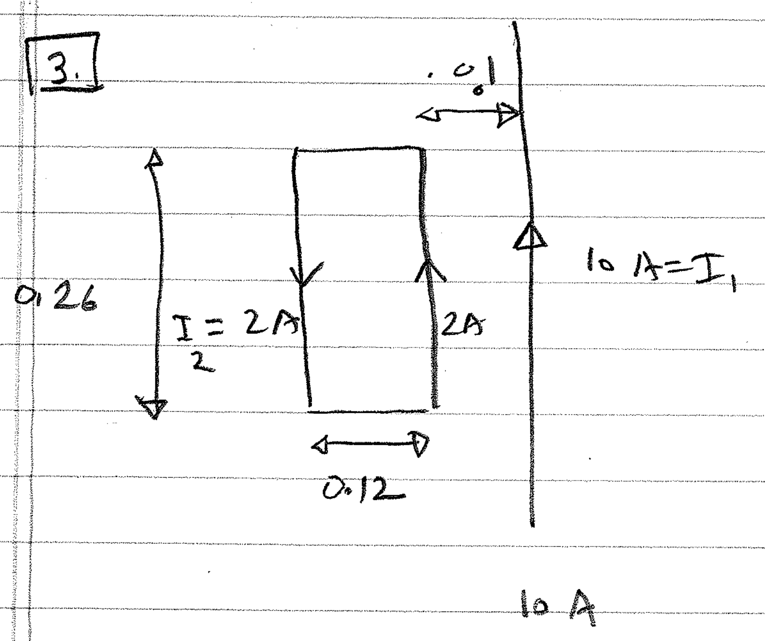
$$R_e = 6.38 \cdot 10^6 \text{ m}$$

$$B = \frac{4\pi \cdot 10^{-7}}{2} \frac{I\,(6.38 \cdot 10^6)^2}{[2(6.38 \cdot 10^6)^2]^{3/2}} = 10^{-4}$$

$$I = \frac{10^{-4}\, 10^{7}}{2\pi} (6.38 \cdot 10^6)\, 2^{3/2}$$

$$= 2.87 \cdot 10^9 \text{ Amps}$$

3.



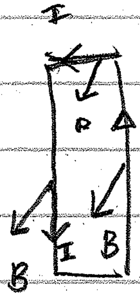
$B = \mu_0 I_1 / 2\pi r$

and $F = I_2 l B$

F is attractive for parallel currents, repulsive for antiparallel

3.

a) For the closer wire $B = \frac{4\pi \cdot 10^{-7}\, 10}{2\pi \cdot (0.1)} = 2 \cdot 10^{-5}$ T



$F = 2(.26)(2 \cdot 10^{-5}) = 1.04 \cdot 10^{-5}$ N

For the fartherwire $B = \frac{4\pi \cdot 10^{-7}\, 10}{2\pi\,(.22)} = 9.1 \cdot 10^{-6}$ N

$F = 2(.26)(9.1 \cdot 10^{-6}) = 4.73 \cdot 10^{-6}$ N

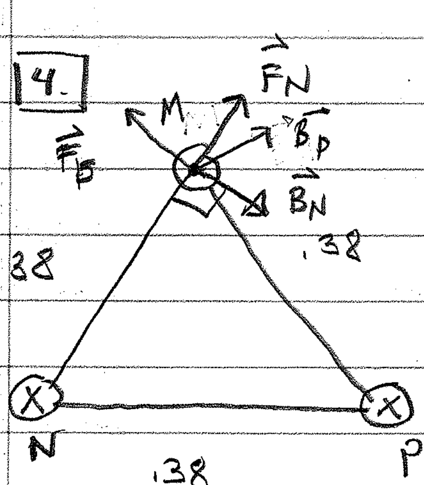
The forces on the horizontal wires cancel

$F_{net} = 9.1 \cdot 10^{-6} - 4.7 \cdot 10^{-6} = 4.4 \cdot 10^{-6}$ to right.

b) There is no torque because all forces are in plane of loop

NB: I will not do the other loop problem since it is basically identical.

$\boxed{4.}$



$I = 8A$

Wire N produces a field B_N at wire M as shown.

The force per length $F_N/L = I_M B_N = I_M \frac{\mu_0 I_N}{2\pi r}$

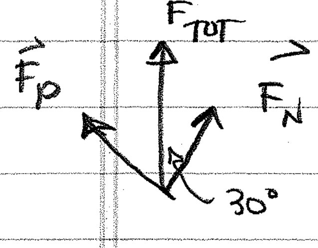
$= 8 \frac{4\pi \cdot 10^{-7}\, 8}{2\pi\,(.38)} = 3.37 \cdot 10^{-5}$ N/m

4.

4 (cont'd) The field $\vec{B}_p$ and force $\vec{F}_p$ are also shown

So the net force per length on wire M is upwards and has

magnitude



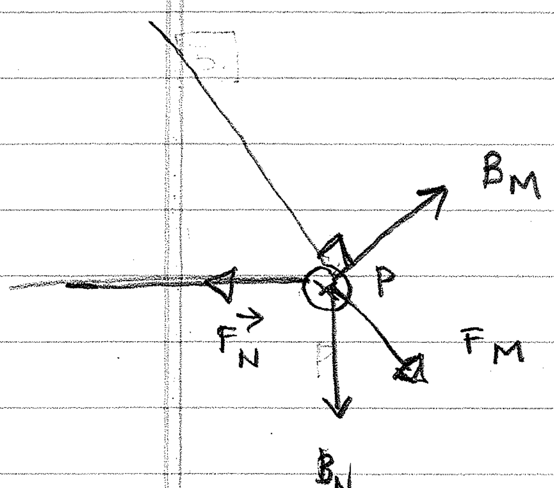
$2(3.37 \cdot 10^{-5}) \cos 30° = 5.84 \cdot 10^{-5}\ N/m$

2 forces due to N and P

only $\hat{y}$ components add

The force per length on P and N can be done similarly. Here are the pictures:

$\frac{\vec{F}_M}{L} =$



5.

$\boxed{5.}$ Use Ampere's law

a) $r < R_1$ $\quad B(2\pi r) = \mu_0 I_0 \frac{\pi r^2}{\pi R_1^2}$

$$B = \frac{\mu_0}{2\pi} I_0 \, r/R_1^2$$

b) $R_1 < R < R_2$ $\quad B(2\pi r) = \mu_0 I \quad B = \frac{\mu_0 I_0}{2\pi r}$

c) $R_2 < R < R_3$ $\quad B(2\pi r) = \mu_0 I_0 - \mu_0 I_0 \frac{\pi(r^2 - R_2^2)}{\pi(R_3^2 - R_2^2)}$

$$= \mu_0 I_0 \left\{ 1 - \frac{r^2 - R_2^2}{R_3^2 - R_2^2} \right\}$$

$$= \mu_0 I_0 \left\{ \frac{R_3^2 - r^2}{R_3^2 - R_2^2} \right\}$$

$$B = \frac{\mu_0 I_0}{2\pi r} \left\{ R_3^2 - r^2 / R_3^2 - R_2^2 \right\}$$

d) $r > R_3$ $\quad B(2\pi r) = 0 \quad B = 0$

6.

$\boxed{6}$ $F = I_2 L B_1$

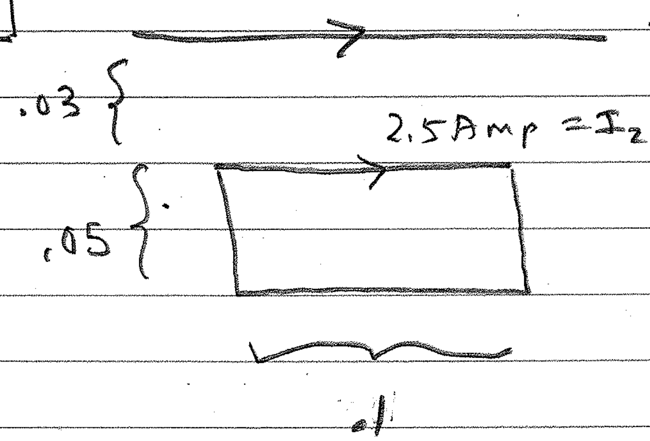
$F/L = I_2 \, \mu_0 I_1 / 2\pi r$

$8.8 \cdot 10^{-4} = I_2 \; 4\pi \cdot 10^{-7} \; 22/2\pi(.07)$

$= I_2 \; 10^{-7} \; \frac{11}{.07}$

$I_2 = 56 \text{ A}$

$\boxed{7}$



B_1 produced by I_1 is into paper.

The forces on vertical pieces of wire loop are equal in magnitude and opposite in direction. They cancel.

The force on closest horizontal piece of loop is

$F = I_2 L B_1 = 2.5\,(.1)\,\frac{\mu_0 I_1}{2\pi r} = (2.5)(.1)\,\frac{2 \cdot 10^{-7}\,(2.5)}{.03}$

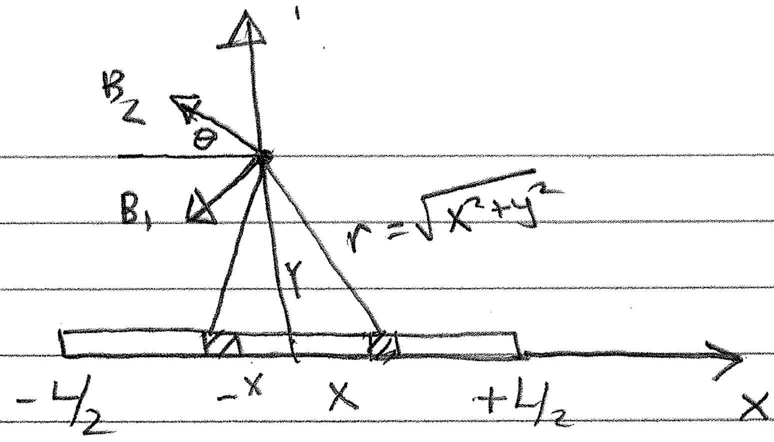
<u>upwards</u> $4.2 \cdot 10^{-6}$ N

on the further section the force is downwards $1.6 \cdot 10^{-6}$ N

Net force is upwards (attractive) $2.6 \cdot 10^{-6}$ N

7.

$\boxed{8.}$



The fields due to pieces of strip at $\pm x$ are shown. ~~Their~~ $\hat{y}$ components cancel, leaving only an $\hat{x}$ component to add up

$$B_x = \int_{-L/2}^{L/2} \underbrace{\frac{I_0\,dx}{L}}_{\substack{\text{current}\\ dI \text{ in piece}\\ \text{of strip}}} \frac{\mu_0}{2\pi\sqrt{x^2+y^2}} \underbrace{\frac{y}{\sqrt{x^2+y^2}}}_{\substack{\cos\theta \text{ for}\\ \hat{x} \text{ component}}}$$

$$= \frac{\mu_0 I_0\, y}{2\pi L}\int_{-L/2}^{L/2} dx \big/ x^2+y^2 \quad \int \frac{d\theta}{y} = \frac{\theta}{y}$$

$x = y\tan\theta$

$dx = y\sec^2\theta\, d\theta$

$x^2+y^2 = y^2\tan^2\theta + y^2 = y^2\sec^2\theta$

$$= \frac{\mu_0 I_0}{2\pi L}\tan^{-1}\frac{x}{y}\Big|_{-L/2}^{L/2} = \frac{\mu_0 I_0}{\pi L}\tan^{-1}\frac{L}{2y}$$

$\boxed{9.}$ a) $F = Eq = \frac{V}{d} q = \frac{10}{.01}(1.6 \cdot 10^{-19})$

$F = 1.6 \cdot 10^{-16}$ N

b) $a = F/m = 1.6 \cdot 10^{-16} / 9.1 \cdot 10^{-31} = 1.76 \cdot 10^{14} \frac{m}{s^2}$

c) $\underbrace{x - x_0}_{.01} = \underbrace{v_0 t}_{0} + \frac{1}{2} a t^2$

$.01 \quad 0 \quad \cdot \frac{1}{2}(1.76 \cdot 10^{14}) t^2$

$t = 1.07 \cdot 10^{-7}$ s