

**Instructions:** This examination is closed book. Only a calculator is allowed. Show all your work. Credit will only be given for *complete* solutions. All questions are worth the same number of points.

**Some facts:** The specific heat of ice is  $2100 \text{ J/(kg K)}$ .

The latent heat of fusion of water is  $334 \times 10^3 \text{ J/kg}$ .

The latent heat of vaporization of water is  $2256 \times 10^3 \text{ J/kg}$ .

The ideal gas law constant  $R=8.314 \text{ J/(mole K)}$ .

1) Three moles of a monatomic ideal gas go through the cycle **abc**. For the complete cycle,  $600 \text{ J}$  of heat flow out of the gas. Process **ab** is at constant pressure, and process **bc** is at constant volume. States **a** and **b** have temperatures  $T_a = 240\text{K}$  and  $T_b = 360\text{K}$ .

- (a) Sketch the  $pV$ -diagram. Explain the reasoning behind the particular cycle trajectory you draw.  
 (b) What is the work  $W$  for the process  $ca$ ?

a) Since process is cyclic  $\Delta U = 0 \rightarrow Q = W$ . If  $600 \text{ J}$  flow out,  $Q = -600 \text{ J}$   
 so  $W = -600 \text{ J}$ . This means the path where  $V$  decreases must be above  
 the path where  $V$  increases. Also, since  $T_b > T_a$  and  $P_b = P_a$ ,  $V_b > V_a$ .



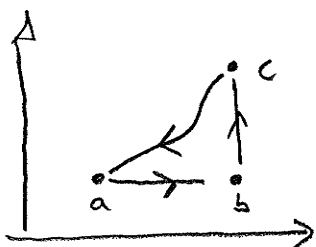
(b)  $W = -600 = W_{ab} + W_{bc} + W_{ca}$   
 $\uparrow \quad \downarrow$   
 since  $\Delta V = 0$

Useful together, but not separately:  $\begin{cases} \Delta U_{ab} = nC_v\Delta T = 3(3/2)(8.314)(120) = 4490 \text{ J} \\ Q_{ab} = nC_p\Delta T = 3(5/2)(8.314)(120) = 7483 \text{ J} \end{cases}$

Since process  $ab$  is not a cycle  $\Delta U_{ab} \neq 0$ . Instead:  $W_{ab} = Q_{ab} - \Delta U_{ab}$

The easier way:  $W_{ab} = p\Delta V = p \frac{nR\Delta T}{P} = nR\Delta T = 3(8.314)(120) = 2993 \text{ J}$

$\Rightarrow W_{ca} = -600 - W_{ab} = -600 - 2993 = \underline{\underline{-3593 \text{ J}}}$



2) An open container holds  $0.750 \text{ kg}$  of ice at  $-20^\circ\text{C}$ . The mass of the container can be ignored. Heat is supplied to the container at a constant rate of  $1100 \text{ J/min}$ .

- (a) After how many minutes does the ice begin to melt?  
 (b) After how many minutes, from the time the heating is first started, does the temperature begin to rise above  $0^\circ\text{C}$ ?

(a) To raise ice  $20^\circ$  from  $-20^\circ$  to  $0^\circ$  requires  
 $Q = mC\Delta T = 0.75(2100)(20) = 31500 \text{ J}$   
 at  $1100 \text{ J/min}$  we need  $28.6 \text{ mins}$

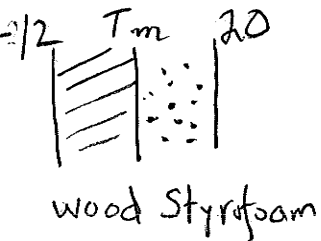
(b) To melt ice requires  $Q = mL_f = (0.75)(334 \cdot 10^3)$   
 $= 250500 \text{ J}$

at  $1100 \text{ J/min}$  this means  $228 \text{ minutes more}$

so  $T$  rises above  $0^\circ\text{C}$  at  $28.6 + 228 \sim 256 \text{ min}$

[3] A carpenter builds an exterior house wall with a layer of wood 4.0 cm thick on the outside, and a layer of Styrofoam 3.0 cm thick on the inside wall surface. The wood has  $k=0.070 \text{ W/(m K)}$  and the Styrofoam has  $k=0.008 \text{ W/(m K)}$ . The interior surface temperature is  $20^\circ\text{C}$  and the exterior surface temperature is  $-12^\circ\text{C}$ . What is the temperature at the plane where the wood meets the Styrofoam? What is the rate of heat flow per square meter through this wall?

(a) Heat flow is  $KA\Delta T/l$ . Assuming transients have settled down the flow through the wood and styrofoam must be equal



$$(0.07) A \frac{(T_m - (-12))}{.04} = \frac{.008 A (20 - T_m)}{.03} \quad \begin{array}{l} (-12^\circ\text{C} = 261\text{K}) \\ (20^\circ\text{C} = 293\text{K}) \end{array}$$

$$1.75 T_m + 21 = 5.33 - 0.27 T_m$$

$$T_m = -7.76^\circ\text{C} \quad (\rightarrow 265.24\text{K})$$

$$(b) \quad \text{Heat flow} = \frac{.008}{.03} A (20 - (-7.76)) = 7.40 A$$

$$\Rightarrow 7.40 \text{ Watts/m}^2$$

[4] At what temperature is the root-mean-square speed of nitrogen molecules equal to the root-mean-square speed of hydrogen molecules at  $30^\circ\text{C}$ ? The molar mass of hydrogen is  $1.008 \text{ g}$  and the molar mass of nitrogen is  $14.007 \text{ g}$ .

$$v_{\text{rms}} = \left( \frac{3}{2} k_B T / m \right)^{1/2}$$

$$\left( \frac{3}{2} k_B T_{\text{N}_2} / m_{\text{N}_2} \right)^{1/2} = \left( \frac{3}{2} k_B T_{\text{H}_2} / m_{\text{H}_2} \right)^{1/2}$$

$$\Rightarrow T_{\text{N}_2} = \frac{m_{\text{N}_2}}{m_{\text{H}_2}} T_{\text{H}_2} = \frac{14.007}{1.008} 303^\circ\text{K}$$

$$= 4210^\circ\text{K} \rightarrow 3937^\circ\text{C}$$

5] Answer two of the following questions. Your answers must be complete and clear. If you respond to more than two, the grader will mark the first ones he sees.

- (a) An ideal gas is expanding (volume increasing). Does the pressure fall faster if the expansion is adiabatic or if it is isothermal. Why?
- (b) Explain the difference between  $C_P$  and  $C_V$ . Which is bigger? Why?
- (c) What is the 'equipartition theorem'? How do you use it to get  $C_V$  for a monatomic gas? What about a diatomic gas?
- (d) What does each of the letters in  $v_{rms}$  stand for? Why do we use  $v_{rms}$  to describe how a bunch of molecules are moving instead of just talking about the average velocity?

(a) In adiabatic expansion no heat enters while the gas does work, hence  $\Delta U < 0$  and  $T$  is falling. So  $T$  is lower for adiabatic expansion than isothermal. A lower pressure results. This can also be seen by comparing eqns  $P = c/v$  (isothermal) and  $P = c/v^\gamma$  ( $\gamma > 1$ ) adiabatic.

(b)  $C_P$  and  $C_V$  are specific heats at constant pressure and volume respectively. For a given rise in temperature  $\Delta T$  more heat is needed if gas expands since it is doing work. Thus at constant  $P$ , where gas expands  $C = \Delta Q / n \Delta T$  is larger than at constant volume.

(c) Every term like  $\frac{1}{2} m v_x^2, \frac{1}{2} m v_y^2, \dots$  to internal energy gives contribution  $\frac{1}{2} k_B$  to specific heat. For monatomic gas we just have  $\frac{1}{2} m (v_x^2 + v_y^2 + v_z^2) \Rightarrow C_V = \frac{3}{2} k_B$ . But diatomic gas has  $\frac{1}{2} k_B$  and  $\frac{1}{2} I \omega^2 \Rightarrow C_V = \frac{5}{2} k_B$ .

(d) rms = root-mean-square. Average velocity of molecule in a gas is zero, so carries no information.

[6] One afternoon while very bored you decide to play a game where you flip a fair coin and move one meter to the right if you get 'heads' and one meter to the left if you get 'tails'. What are the possible distances you can be from your starting point after you toss the coin and move four times? What are the probabilities that you will be at those distances? Explain your reasoning.

There are 16 possibilities:

- TTTT ← 4m to left
- TTHH
- THTH
- THTT
- HTTT } 2 m to left
- TTHH
- THTH
- HTTH
- THTT
- HTTT }  $\phi$  m to left

Possible final positions

-4	-2	$\phi$	2	4
$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

probabilities  
change "distance" to "displacement" or "position"

By symmetry 4 ways to get 3 to 2 to Right and 1 way 4 to right

$v_x < 0$  cancels  $v_x > 0$   
If we square we make everything positive so that when we average to get the mean we get a non-zero result. But we need to take a square root at end to get something with units of velocity



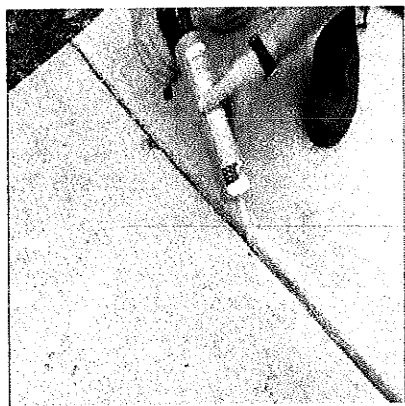
[7] Answer **two** of the following questions. Your answers must be complete and clear. If you respond to more than two, the grader will mark the first ones he sees.

(a) How many Joules are there in a kilowatt-hour? How much does PGE charge you for a kilowatt-hour of electricity? (You will be marked correct if your answer about PGE rates is within a factor of three of the correct one.)

(b) Give two ways temperature is measured. What is the physical property that is changing in each case?

(c) Usually when you add heat to a material its temperature goes up. Name two situations where the temperature goes not increase. Where is the heat energy going?

(d) Why the heck did this numbskull put a long crack in the sidewalk he's building?



$$(a) \text{ kW-hour} = 1000 \frac{\text{J}}{\text{sec}} \cdot 3600 \text{ sec} = 3.6 \cdot 10^6 \text{ J}$$

PGE charges about 20¢ for a kw-hr of electricity

(b) T can be measured with a mercury thermometer.

The volume of mercury changes with T.

T can also be measured by monitoring the change of electrical resistance with T.

(c) When a material is changing from solid to liquid or liquid to gas its temperature stays fixed. The heat goes into breaking bonds between atoms

(d) The builder knows  $\Delta L = L \alpha \Delta T$  so that when T goes up his concrete will expand. Without the expansion joint, it will crack.