Physics 9B, Fall 2013, Midterm 1 KEY

Instructions: This examination is closed book. Only a calculator is allowed. Please show all your work. Credit will only be given for *complete* solutions. All questions are worth the same number of points.

[1] The displacement from equilibrium of a transverse wave traveling on a string is represented by the equation

$$y(x,t) = 1.3\cos(\,5x + 120t\,)$$

y and x are in meters and t is in seconds. What is the wavelength? What is the period? What is the velocity of the wave (magnitude and direction)? What is the amplitude. What is the maximum velocity of a point on the string?

$k = 5 \quad \lambda = 2\pi/k = 2\pi/5 = 1.26\ m$

$\omega = 120 \quad T = 2\pi/\omega = 2\pi/120 = .0524\ s^{-1}\ (\cancel{Hz})$

$v = \omega/k = 120/5 = 24\ m/s$ to <u>left</u>

$A = 1.3\ m$

$v(x,t) = \partial y/\partial t = -\underbrace{120(1.3)}\sin[5x + 120t]$

$v_{max} = 156\ m/s$

[2] The point of a needle on a sewing machine moves in simple harmonic motion along the x axis with frequency 3.4 Hz. At $t = 0$ its position and velocity components are +1.3 cm and -16 cm/s respectively. Write equations giving the position, velocity, and acceleration of the point as functions of time.

$\cancel{f}\ f = 3.4\ s^{-1} \quad \omega = 2\pi f = 21.36$

$x = A\cos\omega t + B\sin\omega t$

$x(0) = 1.3$

$v(0) = -16$

$x(0) = A$

$v = -\omega A\sin\omega t + \omega B\cos\omega t$

$v(0) = \omega B \quad B = v(0)/\omega$

$x(t) = 1.3\cos 3.4t - \underbrace{16/3.4}_{4.71\ .749}\sin 3.4t$

$= 1.5\cos(21.36t + \frac{\pi}{6})$

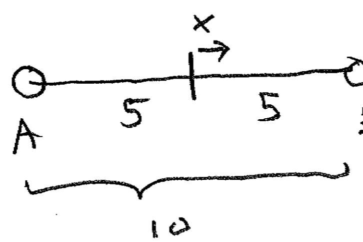
$v(t) = dx/dt = -\underbrace{1.3(3.4)}_{4.42}\sin 3.4t - 16\cos 3.4t \quad \frac{cm}{s}$

$= -32\sin(\quad)$

$= -684\cos(\quad) \quad a(t) = dv/dt = -\underbrace{(4.42)(3.4)}_{15.03\ 593}\cos 3.4t + \overbrace{16(3.4)}^{54.4}_{341}\sin 3.4t \quad \frac{cm}{s^2}$

[3] Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 10 m to the right of speaker A. The frequency emitted by each speaker is 1032 Hz. You are standing between the speakers, along the line connecting them, and are at a point of constructive interference. How far must you walk towards speaker B to move to a point of destructive interference? The speed of sound in air is 344 m/s.

For destructive interference, difference in path length must be $\lambda/2$ (2)



$d = 5 + x - (5 - x) = 2x = \lambda/2$

$x = \lambda/4 \overset{(5)}{} = 1/12 \text{ m} \simeq 0.083 \text{ m}$

$f\lambda = v \quad \lambda = v/f = 344/1032 = 1/3$ (2)

[4] Answer **two** of the following questions. Your answers must be complete and clear. If you respond to more than two, the grader will mark the first ones he sees.

a. Two trombone players in the Aggie marching band are slightly out of tune. One plays a note at frequency 800 Hz and the other plays at 810 Hz. They both play with the same loudness. Describe what the people in the stands will hear.

b. What is Hooke's law? How would you verify that it is correct? (What experiment would you do, what data would you collect, and how would you plot it?)

c. What is the Doppler effect? Give a few examples in 'everyday life' where the Doppler effect is important, and explain briefly how it is used.

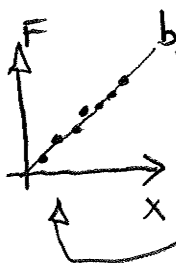
d. What are typical frequencies of AM and FM radio waves? If their speed is 3×10^8 meters/sec, what are their wavelengths?

a) The person in the stands hear a musical tone of 805 Hz, plus an oscillating loudness at 10 Hz. This is the phenomenon of "beats". The loudness oscillations are very slow $T = 2\pi/\omega = 0.63$ sec compared to the tone which has much higher frequency.

Tricky point: sound heard is square of AMP → doubles f to 10

If you like math, $A\cos 800t + A\cos 810t = 2A\cos 805t \cos 5t$.

b) Hooke's law relates the Force a spring exerts on you to how much you stretch it $F = -kx$. To prove Hooke's law, stretch spring to different lengths and plot Force you need



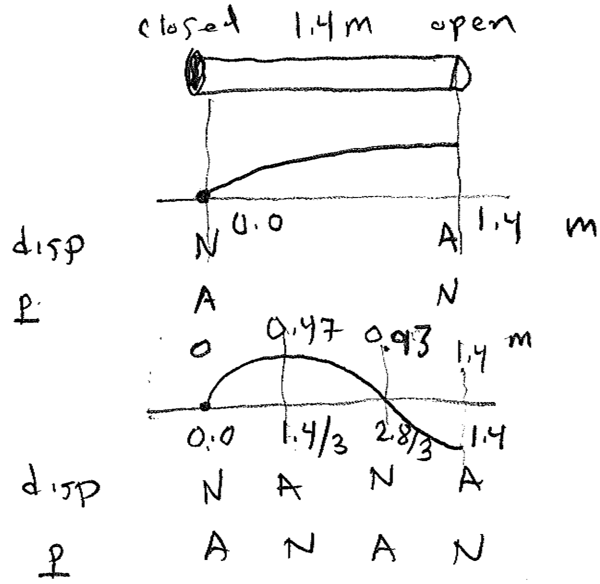
straight line slope k. Notice Force you exert on spring = −F spring exerts on you = +kx

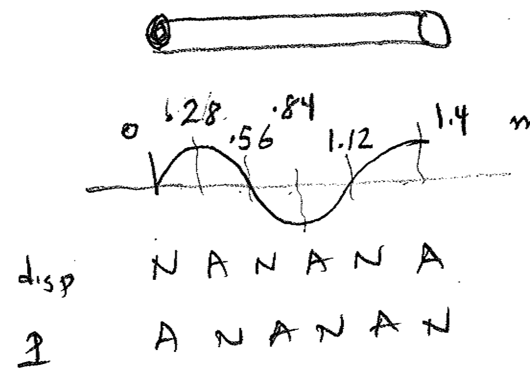
c) When you are moving towards a source of sound (or any wave) you hear a different frequency from when you are not moving. Similarly if source is moving the frequency is shifted. This is called the Doppler Effect. It is used in weather radar and by policeman.

d) FM ~ 100 MHz $= 10^8\ s^{-1}$ $\quad \lambda = v/f = 3$ m

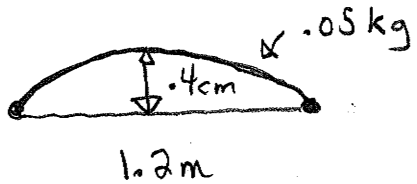
AM ~ 1000 kHz $= 10^6\ s^{-1}$ $\quad = 300$ m

[5] Standing sound waves are produced in a pipe of length 1.4 m. For the fundamental and first two overtones, determine the locations along the pipe (measured from the left end) of the displacement nodes and pressure nodes if the pipe is closed at the left end, and open on the right end. Sketch a typical picture of the displacement as a function of location for the three cases.





[6] A wire of mass 50 g is stretched so that its ends are tied down at points 120 cm apart. The wire vibrates in its fundamental mode with frequency 80 Hz and with an amplitude at the antinode of 0.4 cm. (a) What is the speed of propagation of transverse waves in the wire? (b) Compute the tension in the wire. (c) Find the maximum transverse velocity and acceleration in the wire.



$\frac{1}{2}\lambda = l \qquad \lambda = 2.4\ m$

$f = 80\ s^{-1}$

a) $v = \lambda f = 192\ m/s$

b) $v = \sqrt{T/\mu} \qquad T = \underbrace{\mu}_{M/L} v^2 = \frac{.05}{1.2}(192)^2 = 1536\ N$

c) $v_{max} = \omega x_{max} = 2\pi(80)(\underbrace{.004}_{m}) = 2\ m/s$

d) $a_{max} = \omega^2 x_{max} = [2\pi(80)]^2(.004) = 1011\ \frac{m}{s^2}$

$\gg g\,!$

[7] Answer **two** of the following questions. Your answers must be complete and clear. If you respond to more than two, the grader will mark the first ones he sees.

a. Explain where the equations $\lambda = 2\pi/k$ and $T = 2\pi/\omega$ come from.

b. Show that the sum $\cos 3t + \sin 3t$ can be expressed as $A\cos(3t + \phi)$. What are A and ϕ?

c. The definition of the Bulk modulus $B = -V(dP/dV)$. Derive the equation for B for an ideal gas.

d. The equation for the power in a vibrating string $y(x,t)$ is $P = -T\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$. Where does this equation come from?

(a) Waves are described by $A\cos(kx + \omega t)$ (or sine...)

Cosine repeats every 2π. To get $kx + \omega t$ to change by 2π you must change $\underbrace{x \text{ by } 2\pi/k}$ or $\underbrace{t \text{ by } 2\pi/\omega}$

distance needed to get repeat $\rightarrow$ $\lambda = 2\pi/k$ $\qquad$ $T = 2\pi/\omega$ $\leftarrow$ time needed to get repeat

(b) If $A\cos(3t + \phi) = A(\cos 3t\cos\phi - \sin 3t\sin\phi) = \cos 3t + \sin 3t$

so $A\cos\phi = 1$ and $-A\sin\phi = 1 \Rightarrow \tan\phi = -1$

$\phi = \pi/4$ $\quad$ $\cos\phi = \sqrt{2}/2$

$A = 2/\sqrt{2}$

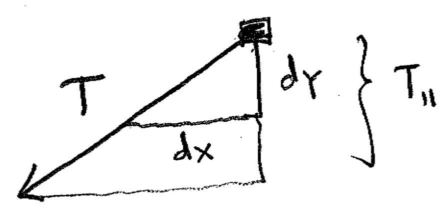
So $\cos 3t + \sin 3t = \frac{2}{\sqrt{2}}\cos(3t - \pi/4)$

(c) $PV = nRT$ $\qquad$ $dP/dV = -nRT/V^2$

$P = nRT/V$ $\qquad$ $B = -V\,dP/dV = nRT/V = P$

d) Power $= F_{\parallel} V$ $\leftarrow$ velocity

$\nearrow$ component of Force parallel to v



similar triangles: $T_{\parallel}/T = -dy/dx$ $\qquad$ $T_{\parallel} = -T\,dy/dx$

$v = dy/dt$ $\quad\Rightarrow\quad$ $P = T_{\parallel}v = -T\,\frac{dy}{dx}\,\frac{dy}{dt}$.

$f = 4.5$

$\omega = 9\pi = 28.27$

$x_0 = 1.7 \quad v_0 = -12$

$\Rightarrow A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = 1.75..$

$\phi = 14° = .245$ rad

$x = 1.75 \cos(28.27 t + .245)$

$= 1.7 \cos \omega t - \frac{12}{\omega} \sin \omega t$

$.42$

$v = -49.5 \, \sin \, (\qquad)$

$= -12 \cos \omega t - 48 \sin \omega t$

$a = -1399 \cos (\qquad)$

$= 339 \sin \qquad - 1356 \cos$

0, 1.6

0, .53, 1.06, 1.6

0, .32, .64, .96, 1.28, 1.6