1.

Physics 9A F2013
Assignment 1

14 – 4, 7, 13, 18, 27, 30, 34, 59, 62, 67, 93

$\boxed{14\text{-}4}$ From Figure we can see $1/2$ of a period occurs starting at $t=1$ and ending at $t=9$. Hence $T = 2(8\text{ sec}) = 16\text{ sec}$.
Then frequency $f = 1/T = 1/16\text{ sec}^{-1}$

Again from Figure the maximum x is 10 cm. So this is the amplitude.

The angular frequency $\omega = 2\pi f = 2\pi/16\text{ sec}^{-1}$
$= 0.393\text{ sec}^{-1}$

$\boxed{14\text{-}7}$ Period $T = 1/f = 1/6\text{ sec}^{-1} = 1/6\text{ sec}$

Angular frequency $\omega = 2\pi f = 12\pi\text{ sec}^{-1}$

$$\omega = \sqrt{\frac{k}{m}} \implies m = k/\omega^2 = 120/(12\pi)^2 = .084\text{ kg}$$

$\boxed{14\text{-}13}$ $x(t) = x_0 \cos\omega t + v_0/\omega \sin\omega t$ $\quad \omega = 2\pi(2.5)$

$$= 1.1\cos 5\pi t - \frac{15}{5\pi}\sin 5\pi t$$

$a(0) = -27.5\pi^2 \dfrac{\text{cm}}{\text{s}^2}$

$$= 1.1\cos 5\pi t - 3/\pi \sin 5\pi t$$

$$v(t) = -5.5\pi\sin 5\pi t - 15\cos 5\pi t$$

$$a(t) = -27.5\pi^2\cos 5\pi t + 75\pi\sin 5\pi t$$

2.

Can also write this as $A\cos(5\pi t+\phi)$ form!

eg $x = 1.46\cos(5\pi t + 0.715)$

14-13 cont'd Can get (a) directly without doing (b) first

$F = ma = -kx \qquad a = -\frac{k}{m}x = -\omega^2 x$

$a = -(2.5(2\pi))^2(1.1) = -27.5\pi^2 \ cm/s^2$

$\boxed{14-18}$ $v = -3.6\frac{cm}{s}\sin((4.71s^{-1})t - \pi/2)$

$\omega = 2\pi f = 2\pi/T$

Given v(t) get x(t) by integrating

$T = 2\pi/4.71 = 1.33$ sec

$x(t) = C + \frac{3.6}{4.71}\cos(4.71t - \pi/2)$

amplitude $A = 0.764$ cm

$a(t) = dv/dt = -3.6(4.71)\cos(4.71t - \pi/2)$

maximum $a = 16.96 \ cm/s^2$

$k = m\omega^2 = 0.5(4.71)^2 = 11.1 \ N/m$

$\boxed{14-27}$ $x(t) = A\cos(\omega t + \phi)$

$v = \frac{dx}{dt} = -\omega A\sin(\omega t+\phi)$

$A = .04$ m

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{450}{.5}} = 30 \sec^{-1}$

a) $v_{max} = 30(.04) = 1.2 \ m/s$

14-27 cont'd

Another way is by energy

$$\frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2$$

$$\frac{1}{2}(450)(.04)^2 = \frac{1}{2}(.5)\, v_{max}^2 \rightarrow v_{max} = 1.2 \text{ m/s}$$

b) $$\frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \text{const} = \frac{1}{2} k A^2$$

$$\frac{1}{2}(.5) v^2 = \frac{1}{2}(450)(.04)^2 - \frac{1}{2} 450(-.015)^2$$

$$\frac{1}{4} v^2 = 0.36 - .0506 \qquad v = 1.112 \text{ m/s}$$

c) $$a = dv/dt = \underbrace{-\omega^2 A}\cos(\omega t + \phi)$$

$$a_{max} = 30^2(.04) = 36 \text{ m/s}^2$$

d) $$-kx = ma \qquad a = -\frac{k}{m} x = -\omega^2 x$$

$$a = -(30)^2(-.015) = 13.5 \text{ m/s}^2$$

e) $$\frac{1}{2} k A^2 = 0.36 \text{ J.}$$

4.

$\boxed{14-30}$ a) $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$= \frac{1}{2}(300)(.012)^2 + \frac{1}{2}(.15)(.3)^2$

$= .0216 + .00675 = .02835\ J$

b) $\frac{1}{2}kA^2 = E \qquad A = \sqrt{2(.02835)/300} = .0137\ m$

c) $\frac{1}{2}mv_{max}^2 = E \qquad v_{max} = \sqrt{2(.02835)/.15} = .615\ m/s$

$\boxed{14-34}$ a) From figure the velocity goes through a full cycle between $t = 0.4$ and $t = 2.0$ sec

$\Rightarrow T = 1.6$ sec

b) $f = 1/T = 1/1.6\ sec^{-1} = 0.625\ sec^{-1}$

c) $\omega = 2\pi f = 2\pi/1.6\ sec^{-1} = 3.925\ sec^{-1}$

d) $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

$A^2 = \frac{m}{k}v_{max}^2 = v_{max}^2/\omega^2 \Rightarrow A = \frac{20}{3.925} = 5.095\ cm$

$v_{max} = 20$ cm/sec from figure

reach amplitude when $v = 0$ eg at $t = 0.4, 1.2, 2.0, \ldots$

14-34 cont'd $\quad -kx = ma$

e) $\quad a_{max} = \frac{k}{m} x_{max} = \omega^2 x_{max} = \omega^2 A$

$$a_{max} = (3.925)^2 (5.095) = 78.5 \ \text{cm}/\text{s}^2$$

acceleration is maximal when X is maximal
which occurs when $v = 0 \Rightarrow t = 0.4, 1.2, 2.0, \ldots$ sec

(f) $\quad k = m\omega^2 \quad m = k/\omega^2 = 75/(3.925)^2 = 4.87$ kg

$\boxed{14\text{-}59}$ $\quad ma = -kx - bv$

a) $\quad m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \qquad x = Ae^{i(\omega t + \phi)}$

$-m\omega^2 = -k - ib\omega \qquad Ae^{i(\omega t+\phi)}$ factors cancel

$$m\omega^2 - ib\omega - k = 0$$

$$\omega = \frac{1}{2m}\left[ib \pm \sqrt{-b^2 + 4km}\right]$$

$$= \frac{ib}{2m} \pm \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

"usual" $\omega = \sqrt{\frac{k}{m}}$ when $b = 0$

When damped

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$ reduced (as expected)

14-59 cont'd

$$w = \sqrt{\frac{2.5}{.3} - \frac{(.9)^2}{4(.3)^2}} = \sqrt{8.333 - 2.25} = 2.47 \text{ sec}^{-1}$$

$$f = w/2\pi = 0.393 \text{ sec}^{-1}$$

b) critically damped $\frac{k}{m} - \frac{b^2}{4m^2} = 0$

$$b = \sqrt{4km} = \sqrt{4(2.5)(.3)} = 1.732 \text{ kg/s}$$

$\boxed{14\text{-}62}$ a) $v = 0$ when $x = \max$ or $\min$

From figure this occurs at $t = 0, 1, 2, 3, 4, \ldots$ sec

b) $E = \frac{1}{2}kA^2 = \frac{1}{2}(225)(.07)^2 = 0.551 \text{ J}$

From figure
(convert to meters)

c)
$$\left.\begin{aligned} E_1 &= \tfrac{1}{2}(225)(.06)^2 \\ E_4 &= \tfrac{1}{2}(225)(.03)^2 \end{aligned}\right\} \quad E_4 - E_1 = -.304 \text{ J}$$

went into heat.

7.

$\boxed{14-67}$ $-kx = ma$

Amplitude

a) $a = \frac{-kx}{m} = -\omega^2 x = -\left(2\pi \frac{4500}{60}\right)^2 .05 = 1.11 \cdot 10^4 \frac{m}{s^2}$

convert to sec
and to angular frequency

b) $F = ma = (.45)(1.11 \cdot 10^4) = 5000\ N$

c) $x = A\cos\omega t$

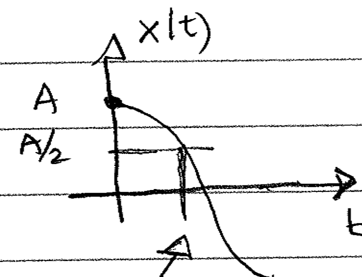
Stroke midpoint $x = \frac{1}{2}A \Rightarrow \cos\omega t = \frac{1}{2}$
$\Rightarrow \sin\omega t = \sqrt{3}/2$

actually by "midpoint" he might mean $x=0$, not $x=\frac{1}{2}A$
In this case set $\sin\omega t = 1$ instead of $\sqrt{3}/2$

$v = -\omega A \sin\omega t$

$= \left(2\pi \frac{4500}{60}\right)(.05)\sqrt{\frac{3}{2}} = 20.4\ m/s$

$KE = \frac{1}{2} m v^2 = 93.7\ J$



d) $P = \frac{\Delta KE}{\Delta t} = \frac{93.7\ J}{.00111} = 84330$ Watts

$1\ HP = 746\ W$ 113 HP

$\omega t = \frac{\pi}{3}$

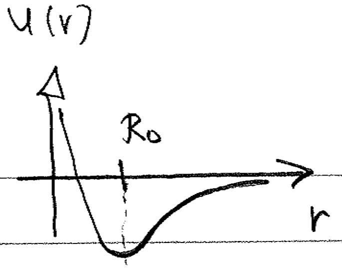
$t = \frac{\pi}{3} \frac{1}{4\pi \frac{4500}{60}}$

$= .00111$ sec

This seems too high le

6 cylinder engine 678 HP ??

$\boxed{14-93}$ $\quad U(r) = A\left[\frac{R_0^7}{8r^8} - \frac{1}{r}\right]$



$R_0 = 2.67 \cdot 10^{-10}\ m \qquad A = 2.31 \cdot 10^{-28}\ J \cdot m$

a) $F = -\frac{dU}{dr} = A\left[\frac{R_0^7}{r^9} - \frac{1}{r^2}\right]$

b) $F = 0$ when $\frac{R_0^7}{r^9} = \frac{1}{r^2} \Rightarrow r = R_0$

c) $U = \min$ when $F = 0$ i.e. $r = R_0$

$$U_{min} = A\left[\frac{R_0^7}{8R_0^8} - \frac{1}{R_0}\right] = -\frac{7}{8}\frac{A}{R_0}$$

$$= -\frac{7}{8}\frac{2.31 \cdot 10^{-28}}{2.67 \cdot 10^{-10}} = -7.57 \cdot 10^{-19}\ J$$

4.73 eV

d)

$$F(R_0 + x) = A\left[\frac{R_0^7}{(R_0+x)^9} - \frac{1}{(R_0+x)^2}\right]$$

$$= A\left[R_0^7 \frac{1}{R_0^9}\left(1+\frac{x}{R_0}\right)^{-9} - \frac{1}{R_0^2}\left(1+\frac{x}{R_0}\right)^{-2}\right]$$

$$= \frac{A}{R_0^2}\left[1 - \frac{9x}{R_0} - \left(1 - \frac{2x}{R_0}\right)\right]$$

$$= -\frac{7A}{R_0^3}\,x \quad \Rightarrow \quad k = +7A/R_0^3$$

9.

$$k = \frac{7(2.31 \cdot 10^{-28})}{(2.67 \cdot 10^{-10})^3} = 84.95$$

14-93 cont'd

$$\omega = \sqrt{\frac{k}{m}} = 5.27 \cdot 10^{13} \text{ sec}^{-1} \qquad f = \frac{\omega}{2\pi} = 8.39 \cdot 10^{12} \text{ sec}^{-1}$$

where does $3.06 \cdot 10^{-26}$ come from

K $\quad m_1 = \quad (1.67 \cdot 10^{-27})$

Cl $\quad m_2 = \quad (1.67 \cdot 10^{-27})$

mass of proton/neutron

Atomic number

What color light does this f correspond to ?

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{8.39 \cdot 10^{12}} = 3.58 \cdot 10^{-5} \text{ m}$$

not visible