MIDTERM EXAM: Physics 9A, Summer 2015

General Instructions/Information: This quiz is closed book. A calculator is allowed. Please show all your work, and give units for all answers and on all graphs. Label your graphs with numbers whenever possible. Credit will only be given for complete solutions.

[1.] (20 points) A 8.0 kg box moving at 5.0 m/s on a horizontal frictionless surface runs into a light spring of force constant $k = 200$ N/m. Use the work-energy theorem to find the maximum compression of the spring.

\[
\text{Work done} = \text{change in KE} = \frac{1}{2} k (x_1^2 - x_2^2) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2
\]

\[
\Rightarrow \frac{1}{2} 200 x_2^2 = \frac{1}{2} 8 \times 5^2
\]

\[
x_2 = 1 \text{ m}
\]

[2.] (30 points) A person stands at the base of a hill that is a straight incline making an angle $\phi$ with the horizontal. (See figure.) A ball is thrown with initial speed $v_0$ at an angle $\alpha$ with the horizontal. What are the $x$ and $y$ coordinates of the ball as a function of time? At what time $t$ will the ball hit the hill?

\[
x(t) = v_0 \cos \alpha \cdot t
\]

\[
y(t) = v_0 \sin \alpha \cdot t - \frac{1}{2} g t^2
\]

The hill is specified by

\[
y/x = \tan \phi \Rightarrow y = x \tan \phi
\]

The ball hits the hill when this condition is satisfied

\[
v_0 \sin \alpha t - \frac{1}{2} g t^2 = v_0 \cos \alpha t \tan \phi
\]

\[
v_0 \sin \alpha - v_0 \cos \alpha \tan \phi = \frac{1}{2} g t
\]

\[
t = \frac{2v_0}{g} \left\{ \sin \alpha - \cos \alpha \tan \phi \right\}
\]

Can also write as

\[
t = \frac{2v_0}{g} \cos \alpha \left\{ \tan \alpha - \tan \phi \right\}
\]

which makes it clear we need $\alpha > \phi$ for the problem to be meaningful ($t > 0$).
(3.) (30 points) A 6.0 kg package slides 2.0 m down a long ramp that is inclined at 30° below the horizontal. The coefficient of kinetic friction between the package and the ramp is \( \mu_k = 0.20 \). Calculate (b) the work done on the package by friction; (c) the work done on the package by gravity; (d) the work done on the package by the normal force; (e) the total work done on the package; (f) if the package has a speed 2.5 m/s at the top of the ramp, what is its speed after sliding 2.0 m down the ramp?

(b) \[ W_{\text{fric}} = \int F_{\text{fric}} \cdot ds = \mu_k N \cdot ds = \mu_k mg \cos \theta = 0.20 \times 6.0 \times 9.8 \times \cos 30° = 10.2 \text{ N} \]

(c) \[ W_{\text{grav}} = \int F_{\text{grav}} \cdot ds = mg \cos \theta \cdot ds = 6 \times 9.8 \times 2 \cos 30° = 58.8 \text{ J} \]

(d) \[ W_N = 0 \text{ since } \theta = 90° \]

(e) \[ W_{\text{TOT}} = 38.4 \text{ J} \]

(f) **Work-Energy Theorem**
\[ 38.4 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \rightarrow v_f = 6.68 \text{ m/s} \]

(4.) (10 points) The figure shows the position of an object as a function of time. At each point time \( t_1, t_2, t_3, t_4, \) and \( t_5 \), indicate whether the total force on the object is positive, negative, or zero.  

\[ \text{Since } x(t) \text{ is linear at } t_1, t_3, t_5 \]

\[ \text{The velocity } v \text{ is constant. This means the acceleration } \frac{dv}{dt} = a \]

\[ \text{is zero. So } F = ma = 0.} \]

\[ \text{At } t_2 \text{ the slope of } x(t) \text{ is increasing. So velocity increasing, so acceleration is positive. Force must be positive.} \]

\[ \text{Similarly at } t_4 \text{ slope decreasing } \rightarrow v \text{ decreasing } \rightarrow a < 0 \rightarrow F < 0. \]
[5.] (20 points) A large wrecking ball is held in place by two light steel cables. See figure. If the mass \( m \) of the wrecking ball is 5000 kg, what are (a) the tension \( T_B \) in the cable that makes an angle of \( \theta = 30^\circ \) with the vertical; and (b) the tension \( T_A \) in the horizontal cable?

\[
\text{Since ball not moving } \sum \vec{F} = 0
\]

\[
\text{In } x \text{ direction:} \quad -T_A + T_B \cos(90-\theta) = 0
\]

\[
-T_A + T_B \sin \theta = 0
\]

\[
\text{In } y \text{ direction} \quad T_B \sin(90-\theta) - Mg = 0
\]

\[
T_B = \frac{Mg}{\cos \theta} = \frac{5000 \text{ kg}(9.8 \text{ m/s}^2)}{\cos 30^\circ} = 56580 \text{ N}
\]

\[
T_A = T_B \sin 30 = 28290 \text{ N}
\]

[6.] (20 points) In one of his experiments, Galileo noticed that the ratio of the acceleration of a ball rolling down an incline to the acceleration of a ball in free fall is equal to the ratio of the height of the incline to its length. Show that this is true.

\[
\text{We know } a = g \sin \theta \text{ for ball going down incline}
\]

\[
\text{But } \sin \theta = \frac{h}{l} \Rightarrow \frac{a}{g} = \sin \theta = \frac{h}{l}
\]

[7.] (10 points) In one of the “Harry Potter” books, Ron Weasley is speaking with excitement about his new broomstick: “It can go from 0 to 60 miles per hour in ten seconds.” Name the property of the broomstick he is describing, and give a one sentence explanation of your answer.

\[
\text{acceleration measures } \frac{\text{d}v}{\text{d}t} = 60 \text{ mph/10 sec}
\]

So Ron is speaking of acceleration.
[8.] (30 points) (a) Sketch the position, velocity, and acceleration versus time for a ball dropped from rest \((v_0 = 0)\) off a tall \((h=400 \text{ m})\) building if there is no air resistance. Choose your origin for \(y\) to be at the base of the building, so that \(y_0 = h\), and choose the positive \(y\) direction to be upwards. Label your axes with appropriate numerical values.

(b) In class we discussed what happens when considering the friction of an object falling through the air. On what variable does such a friction force depend? Sketch the position, velocity, and acceleration if air resistance is present, and large enough so that its effects are clearly evident.

\[
400 = \frac{1}{2} (9.8) t^2 \rightarrow t = 9.03 \text{ sec} \quad \text{time to fall}
\]

\[V = v_0 + at\]
\[= 0 - 9.8(9.03)\]
\[= -88.5 \text{ m/s}\]

\[a = \frac{dv}{dt} = -9.8 \text{ m/s}^2\]

\[F_{\text{air, resistance}} = -CV\]

Falling object reaches terminal velocity \(mg = CVt\) instead of accelerating forever.
[9.] (10 points) Extra Credit: Steel makers deliberately mix a very small amount of carbon into iron. In a couple of sentences, explain why reducing the purity of iron actually make it stronger.

Carbon defects provide places for cracks to start and not go all the way through material.

[10.] (30 points) The position \( \vec{r} = x(t) \hat{i} + y(t) \hat{j} \) of an object as a function of time obeys

\[
x(t) = 6 \cos 2t \quad \quad y(t) = 6 \sin 2t
\]

(a) Show the object’s path is a circle. What is the radius \( r \)?
(b) Show that the speed (the magnitude of the velocity vector \( \vec{v} \)) is constant.
(c) Show that the acceleration obeys \( |\vec{a}| = |\vec{v}|^2 / r \)
(d) Extra Credit (10 points) Give formulae for \( x(t) \) and \( y(t) \) which would still give circular motion, but with a speed which is not constant. Explain.

\[
(a) |\vec{r}|^2 = x^2 + y^2 = 36 \cos^2 2t + 36 \sin^2 2t = 36 = r^2
\]

This is eqn of circle of radius \( r = 6 \) m

\[
(b) \quad v_x = -12 \sin 2t \quad \quad v_y = 12 \cos 2t
\]

\[
|\vec{v}|^2 = v_x^2 + v_y^2 = 144 \sin^2 2t + 144 \cos^2 2t = 144
\]

\[
|\vec{v}| = 12 \text{ m/s}
\]

\[
(c) \quad a_x = -24 \cos 2t \quad \quad a_y = -24 \sin 2t
\]

\[
|\vec{a}| = 24 \quad \text{by same calculations as in a) b)}
\]

\[
|\vec{v}|^2 / r = 12^2 / 6 = 24
\]

\[
(d) \quad x(t) = 6 \cos 2t^2 \quad \quad y(t) = 6 \sin 2t^2
\]

Still have \( x^2 + y^2 = 36 \) so circle

but \( |\vec{v}| = 24t \) is no longer constant