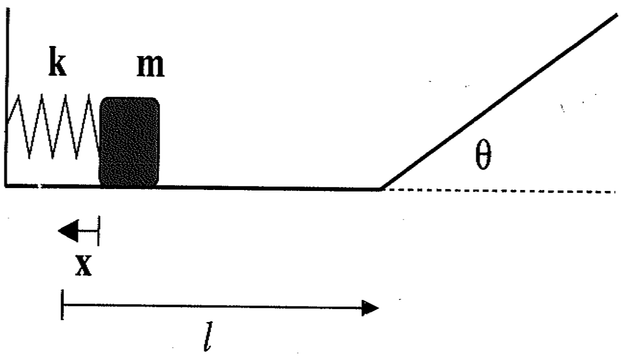
NAME: KEY

MIDTERM 2 (v1): Physics 9A, Spring 2012

General Instructions/Information: This exam is closed book. Only a calculator is allowed. Please show all your work, and give units for all answers and on all graphs. Credit will only be given for complete solutions. The acceleration of gravity is g=9.8 m/s^2 downwards.

[1.] (25 points) A 3.00 kg block is pushed against a spring with negligible mass and force constant $k = 500$ N/m, compressing it $x = 0.31$ m. When the block is released, it moves along a frictionless horizontal surface, and then up a frictionless ramp with slope $\theta = 30°$. (a) What is the speed of the block as it slides along the horizontal surface after having left the spring? (b) How far does the block travel up the incline before starting to slide back down? (c) Suppose instead that there is friction $\mu_k = 0.10$ along the horizontal surface only (still no friction on the incline). If the block travels $l = 3$ m along the horizontal surface before starting up the incline, how is the distance it goes up the incline changed?



a) From E conservation

$$\tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 + mgy = \text{Const}$$

$\tfrac{1}{2}500(.31)^2 + 0 + 0$ ← initial

$0 + \tfrac{1}{2}mv^2 + 0$ ← along horiz surface after leaving spring

So $\tfrac{1}{2}3v^2 = \tfrac{1}{2}500(.31)^2$

$v = 4.00$ m/s

b) Again from E conservation, at max height

$\tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 + mgy$

$0 + 0 + mgy$

So $3(9.8)y = \tfrac{1}{2}500(.31)^2$ $y = .81$ m

Distance along incline $= 2y$ if want to give answer that way

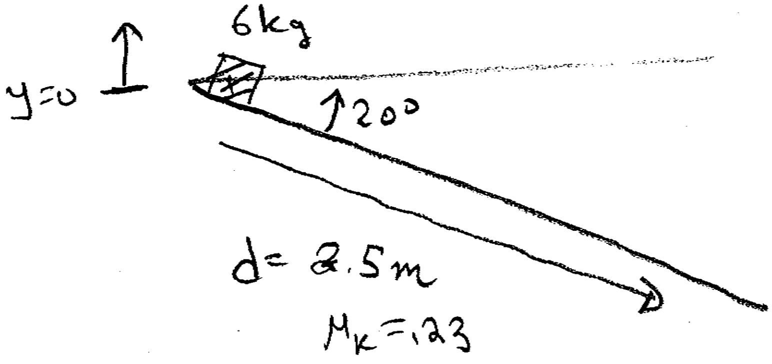
c) If there is friction the block loses energy (work done by friction < 0) $W_{fric} = -F_{fric}l = -N\mu_k l$

$= -3(9.8)(.1)(3) = -8.82$ J

so now $mgy = \tfrac{1}{2}500(.31)^2 - 8.82 = 24.02 - 8.82$

$y = 0.52$ m

[2.] (25 points) A 6.00 kg package slides 2.50 m down a long ramp that is inclined 20° below the horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.23$. Calculate (a) the work done on the package by friction; (b) the work done on the package by gravity; (c) the work done on the package by the normal force; (d) the total work done on the package. (e) If the package has a speed 3.00 m/s at the top of the ramp, what is its speed after sliding 2.50 m down the ramp?



a) $W_{fric} = F_{fric}\, d \cos\pi = N\mu_k d \cos\pi$

$= mg\cos 20\ \mu_k\, d\,(-1)$

$= -6(9.8)\cos 20\,(.23)(2.5) = -31.8\text{ J}$

b) $W_{grav} = -mg(Y_f - Y_i)$

$= -mg(-2.5\sin 20 - 0)$

$= 6(9.8)(2.5)\sin 20 = +50.3\text{ J}$

c) $W_N = 0$

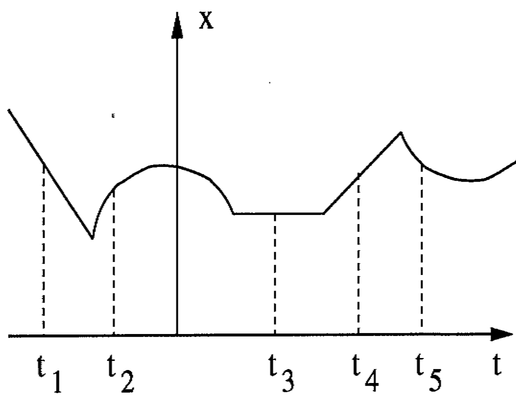
d) $W_{TOT} = 50.3 - 31.8 = 18.5\text{ J}$

e) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{TOT}$

$\frac{1}{2}6\,v_f^2 = \frac{1}{2}6(3)^2 + 18.5$

$v_f = 3.89\text{ m/s}$

[3.] (10 points) The figure shows the position of an object as a function of time. At each point time t_1, t_2, t_3, t_4, and t_5, indicate whether the total force on the object is positive, negative, or zero. Briefly explain your reasoning.



At t_1, t_3, and t_4

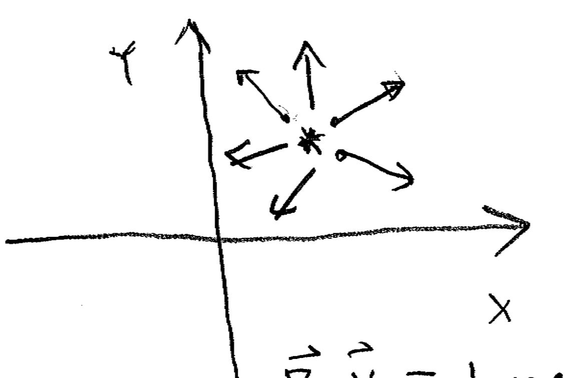
$V = dx/dt = \text{const}$ so $a = \frac{d^2x}{dt^2} = 0$

Thus $F = 0$,

At t_2 $\frac{d^2x}{dt^2} < 0$ so $F < 0$

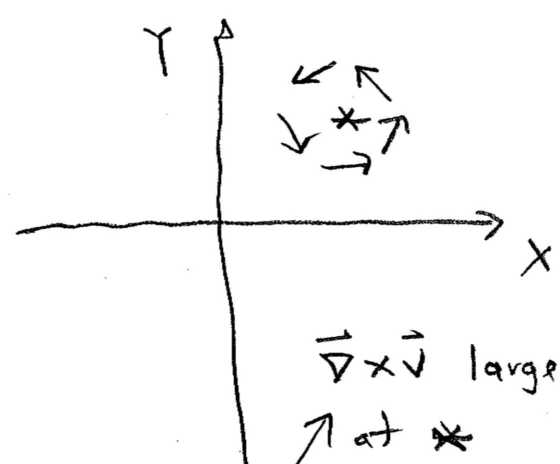
At t_5 $\frac{d^2x}{dt^2} > 0$ so $F > 0$

[4.] (10 points) In conventional, single variable, calculus we assign an 'output' number $f(x)$ to an 'input' number x. In vector calculus we instead assign a vector $\vec{v} = v_x\hat{i} + v_y\hat{j}$ to several input numbers x, y. To do calculus we then define generalizations of the derivative df/dx. Two of these are the 'divergence' and the 'curl'. Draw a collection of vectors $\vec{v}$ on the $x-y$ plane which will have a large divergence. Draw a collection of vectors $\vec{v}$ on the $x-y$ plane which will have a large curl.



$\vec{\nabla} \cdot \vec{v}$ = large at $*$

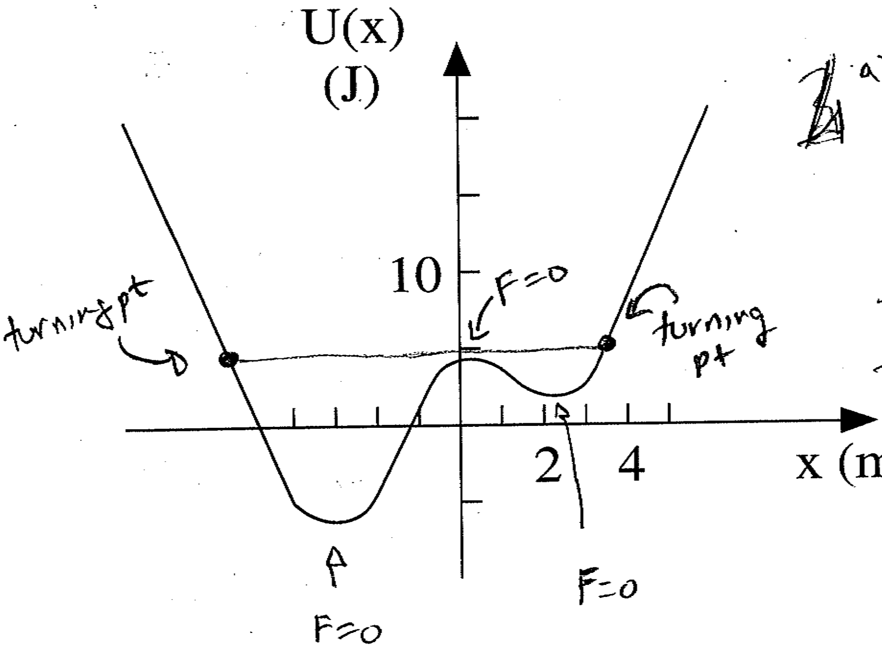
Divergence



$\vec{\nabla} \times \vec{v}$ large at $*$

curl

[5.] **(15 points)** The figure below shows the potential energy $U(x)$ as a function of position x. (a) Mark the positions where the force is zero. (b) What is the direction of the force at $x = 4$? (c) If an object moving in this potential energy has a total energy of 5 Joules, mark the turning points of its motion. (d) Again, assuming the total energy is 5 J, estimate the object's kinetic energy at points $x = -3$ and $x = 2$.



4 a) $F(x) = -\frac{du}{dx}$

so F = 0 where u has max or min

3 b) Force at $x = 4$ is in $\ominus$ x direction since $\frac{du}{dx} > 0$

4 c) turn at $u(x) = E$

4 d) $KE + u(x) = 5$

$KE = 5 - u(x)$

at $x = -3$ $u(x) \sim -6$ so $KE = 11$ J

at $x = 2$ $u(x) \sim 2$ so $KE = 3$ J

[6.] **(15 points)** An object of mass $m = 4$ kg moves along the x axis with a force $F(x) = 5x^2 - 2x$. It has velocity $v = 3$ m/s at $x = 1$. What is its velocity at $x = 2$?

$$W = \int_1^2 F(x)dx = \int_1^2 (5x^2 - 2x)\,dx = \frac{5}{3}x^3 - x^2 \Big|_1^2$$

$$= \left(\frac{40}{3} - 4\right) - \left(\frac{5}{3} - 1\right) = \frac{35}{3} - 3 = \frac{26}{3}$$

$W = \Delta KE$

$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 + W$

$\frac{1}{2} 4 v_f^2 = \frac{1}{2} 4 (3)^2 + \frac{26}{3} = 18 + \frac{26}{3} =$

$v_f = 3.65$ m/s