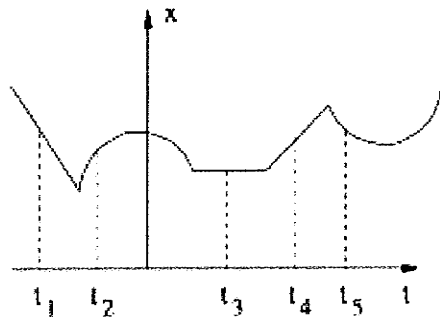


There are eight problems on seven pages none should be blank. Except for problem #1, no credit will be given for answers without work. Answers must have correct units. Remember that the grader must be able to understand your solution to give you partial credit; therefore, incomprehensible work will receive no partial credit. Good Luck!

[1] Circle the answer that correctly completes the sentence correctly.

- (a) A truck is accelerated in the forward direction by the force of the (truck's tire on the ground | ground on the truck's tire | none of the above).
- (b) A rough surface can exert a(n) (normal | tangential | all of the above | none of the above) force.
- (c) Many of the great rivers in the world have a tendency to flow from the pole towards the equator. Over time as they carry sediment towards the equator, the length of the earth's day will (increase | decrease | not change).
- (d) For the graph of the potential energy of a particle as a function of x , the particle will have the greatest speed where the potential energy graph is (maximum | | steepest | | minimum | none of the above)
- (e) For an object in circular motion (centripetal | tangential | all of the above | none of the above) acceleration will change the direction of the velocity vector.

Statements (f) through (h) refer to the figure below which shows the position of an object as a function of time. Circle all the times that complete the statement correctly



- (f) The total force acting on the object is positive at (t_1 | t_2 | t_3 | t_4 | t_5)
- (g) The total force acting on the object is negative at (t_1 | t_2 | t_3 | t_4 | t_5).
- (h) The total force acting on the object is zero (t_1 | t_2 | t_3 | t_4 | t_5).

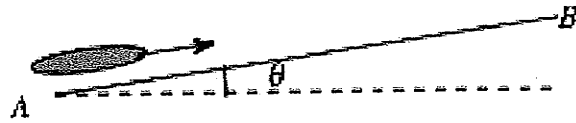
Constants:

$$g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

1	
2	
3	
4	
5	
6	
7	
8	
Σ	

[2] A 90,000 kg jet has two engines which produce a constant thrust of 110,000 N each during the takeoff roll along the runway. The takeoff speed is 210 km/hr. Neglect air and rolling resistance, and assume the engine thrust is parallel to the runway. The runway is slightly inclined at $\theta = 1^\circ$.



(a) Determine the length s of the runway required first for an uphill takeoff direction from A to B ,

$$\underbrace{2(110000)}_{\text{Thrust}} - \underbrace{90000(9.8)\sin\theta}_{\substack{\text{component of gravity} \\ \text{down runway}}} = 90000 a$$

$$\Rightarrow a = 2.273 \text{ m/s}^2$$

$$v^2 - \underbrace{v_0^2}_{\neq} = 2a(\underbrace{x-x_0}_{\text{runway length}})$$

$$x-x_0 = [210(1000)/3600]^2 / 2(2.273) = 748 \text{ m}$$

(b) and second determine the length s for a downhill takeoff direction from B to A .

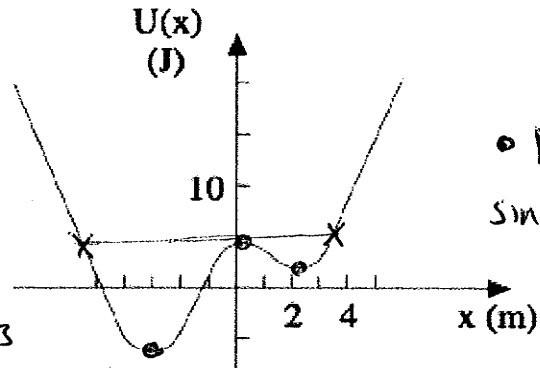
Same eqn but change sign of gravity term

$$x-x_0 = 650.5 \text{ m}$$

[3] The figure below shows the potential energy $U(x)$ as a function of the position x .

- (a) Mark the positions where the force is zero.
(b) What is the direction of the force at $x = 4$ m?

to the left (-x direction)



- (c) If an object moving in this potential energy has a total energy of 5 Joules, mark the turning points of its motion. $x = \text{turning points}$
(d) Assuming the total energy is 5 J, estimate the object's kinetic energy at points $x = -3$ m and $x = 2$ m.

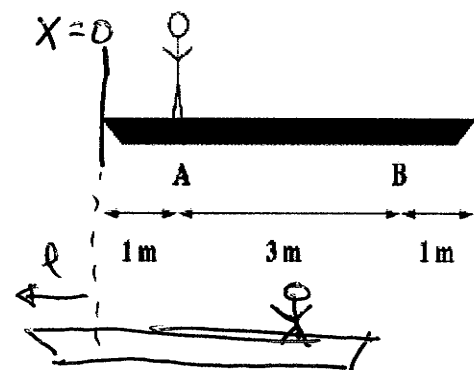
$$PE + KE = E \quad KE = E - PE = 5 - (-6) \approx 11 \text{ at } x = -3$$

$$= 5 - (2) \approx 3 \text{ at } x = 2$$

[4] A 50 kg woman stands up in an 80 kg canoe 5.00 m long. She walks from a point A which is 1.00 m from one end to a point B which is 1.00 m from the other end. If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?

initially $x_{cm} = \frac{50(1) + 80(2.5)}{50 + 80}$

$$= \frac{250}{130}$$



finally $x_{cm} = \frac{50(4-l) + 80(2.5-l)}{130} = 400 - 130l$

$$400 - 130l = 250 \quad \text{since CM position unchanged (no external forces)}$$

$$130l = 150$$

$$l = 15/13 \text{ m} \quad \text{Page 3 of 7}$$

[5] A children's merry-go-round consists of a thin disk of mass $2m$ and radius r . It is initially spinning with angular speed ω_i . There is a thin girl of mass m standing at the outer edge of the merry-go-round. She walks along a radial path to the center of the merry-go-round. (a) Explain what happens to the girl-merry-go-round system. The moment of inertia of a disk is $I = \frac{1}{2}Mr^2$

The angular velocity increases because the moment of inertia decreases as mass moves closer to axis. ω must increase to keep angular momentum $L = I\omega$ constant

(b) What is the final angular speed of the merry-go-round?

$$I_i \omega_i = I_f \omega_f$$

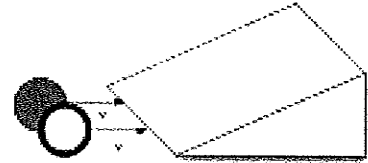
$$\left[\underbrace{\frac{1}{2}(2m)(r^2)}_{\text{merry go round}} + \underbrace{m r^2}_{\text{girl at edge}} \right] \omega_i = \left[\underbrace{\frac{1}{2}(2m)(r^2)}_{\text{merry go round}} + \underbrace{0}_{\text{girl at center } r=0} \right] \omega_f$$

$$2mr^2 \omega_i = mr^2 \omega_f$$

$$\boxed{\omega_f = 2\omega_i}$$

[6] The moment of inertia of a solid sphere is $I_s = \frac{2}{5}mr^2$. The moment of inertia of a ring is $I_r = mr^2$. A ~~disk~~ ^{sphere} and a ring with equal mass (m) and equal radii, r , both roll up an inclined plane. They start with the same linear velocity, v , for the center of mass.

(a) Clearly explain which will go higher.



The ring has a larger I and hence more rotational kinetic energy (if its ω is same as sphere) ~~and also its~~ Its translational k.e. of center of mass is same as sphere. All in all it has more KE and hence can get to higher spot on ramp ^{sphere}

(b) Use conservation of energy to determine the maximum vertical height, h_d , the ~~disk~~ and the maximum vertical height, h_r , the ring will reach.

$$\frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 = mgh \quad \omega = v/r$$

Ring $\frac{1}{2} M R^2 \omega^2 + \frac{1}{2} M v^2 = mgh$

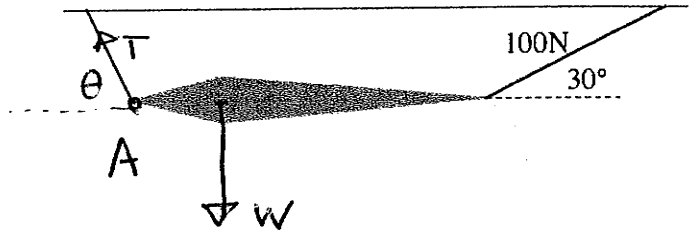
$$M v^2 = mgh \quad h_r = \frac{v^2}{g}$$

Sphere $\frac{1}{2} \frac{2}{5} M R^2 \omega^2 + \frac{1}{2} M v^2 = mgh$

$$\frac{7}{10} M v^2 = mgh \quad h_s = \frac{7v^2}{10g}$$

as explained in (a) $h_r > h_s$.

[7] An art object of nonuniform composition is suspended horizontally from the ceiling by two wires at its ends. The object is of length L , and its center of mass is $L/4$ from its left end. The wire at its right end makes a 30° angle with the horizontal and has a 100 N tension. Determine the object's weight and the tension in the other wire.



$$\sum F_x = 0 = 100 \cos 30 - T \cos \theta = 0$$

$$\sum F_y = 0 = T \sin \theta + 100 \sin 30 - W = 0$$

$$\sum \tau = 0 = -W \frac{L}{4} + 100 L \sin 30 = 0$$

(about A)

torque eqn gives $W = 400 \sin 30 = 200\text{ N}$

$$x: T \cos \theta = 100 \cos 30$$

$$y: T \sin \theta = W - 100 \sin 30 = 200 - 50 = 150$$

$$T \cos \theta = 86.6$$

$$T \sin \theta = 150$$

$$T^2 = (86.6)^2 + (150)^2 \Rightarrow T = 173\text{ N}$$

can also compute θ

$$\tan \theta = 150/86.6 \Rightarrow \theta = 60^\circ$$

[8] The density of a typical asteroid is $2.5 \times 10^3 \text{ kg/m}^3$. This is important, for you're intent on finding the largest heavenly body on which you can stand and launch a rock into a low circular orbit *by hand*, and your best throw is only 40 m/s.

(a) Assuming a spherical asteroid (not a very realistic assumption), what is the radius of the asteroid you desire? (Note: "low orbit" means the minimum possible radius.)

The volume of a sphere is $\frac{4}{3}\pi R^3$.)

$$F_{\text{grav}} = mv^2/r$$

$$G Mm/r^2 = mv^2/r \quad \text{But also } M = \frac{4}{3}\pi r^3 \rho$$

$$\text{So } M = \frac{v^2 r}{G} = \frac{4}{3}\pi r^3 \rho$$

$$r^2 = \frac{3v^2}{4\pi G \rho} = 4.79 \cdot 10^4 \text{ m}$$

(b) If you threw the rock instead straight up, would it go forever, and if not, how high would it go? Assume the radius is for the asteroid in part (a).

You can solve by plugging in numbers. A better way is to observe

$$G Mm/r^2 = mv^2/r \quad \text{for circular orbit}$$

$$\Rightarrow G Mm/r = mv^2$$

Thus the Energy is

$$E = -\frac{G Mm}{r} + \frac{1}{2}mv^2 = -\frac{G Mm}{r} + \frac{1}{2}\frac{G Mm}{r} = -\frac{1}{2}\frac{G Mm}{r}$$

Since $E < 0$ rock is bound to asteroid. The final

$$\text{height is } -\frac{G Mm}{r_f} + \underbrace{\phi}_A = -\frac{1}{2}\frac{G Mm}{r}$$