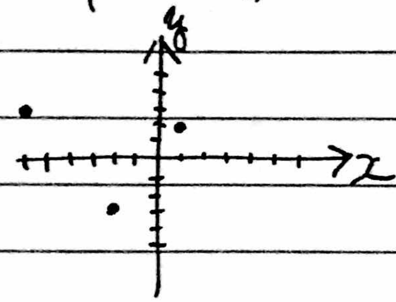
1. PHY9A Discussion 9 May 29, 2018



1.

i) $I_x = \sum_i m_i y_i^2 = m(2^2+3^2+(-3)^2)$

$= 22m.$

$I_y = \sum_i m_i x_i^2 = m(1^2+(6)^2+(2)^2)$

$= 41m.$

ii) $I_z = \sum_i m_i r_i^2 = \sum_i m_i(x_i^2+y_i^2)$

$= m(1^2+2^2+(-6)^2+3^2+(-2)^2+(-3)^2) = 63m$

$= I_x + I_y.$

iii) Let (x, y) the location of the axis.

$I(x,y) = \sum_i m_i[(x_i-x)^2+(y_i-y)^2]$

$= \sum_i m_i[x^2-2xx_i+x_i^2+y^2-2yy_i+y_i^2]$

$= 3mx^2 - 2mx\sum_i x_i + 3my^2 - 2my\sum_i y_i + \sum_i m_i(x_i^2+y_i^2)$

$= 3mx^2 + 14mx + 3my^2 - 4my + 63m$

$= 3m(x^2+2\cdot\frac{7}{3}x) + 3m(y^2-2\cdot\frac{2}{3}y) + 63m$

$= 3m(x+\frac{7}{3})^2 - \frac{49}{3}m + 3m(y-\frac{2}{3})^2 - \frac{4}{3}m + 63m$

2.

$$\therefore\ I(x,y) = 3m\left(x+\frac{7}{3}\right)^2 + 3m\left(y-\frac{2}{3}\right)^2 + \frac{136}{3}m.$$

$I(x,y)$ is minimized at $(x,y)=\left(-\frac{7}{3},\frac{2}{3}\right)$ (the center of mass) and becomes $I_{com} = \frac{136}{3}m \approx 45.3\times m$.

2.

i) Since it's a uniform disk, $\sigma = \frac{m}{\pi R^2}$, and

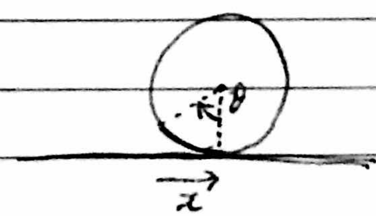
$$I = \int r^2 dm = \int r^2 \frac{dm}{ds} ds = \int \sigma r^2 \cdot r\,dr\,d\varphi$$

$$= \sigma \cdot \int_0^{2\pi} d\varphi \int_0^R r^3 dr = \frac{m}{\pi R^2}\cdot 2\pi \cdot \frac{R^4}{4}$$

$$= \frac{1}{2}mR^2 = \frac{1}{2}\cdot 5\cdot\left(\frac{1}{2}\right)^2 = 0.625\ [\mathrm{kg\cdot m^2}]$$

ii) $(-\theta)\cdot R = x$

$\rightsquigarrow (-\omega)\cdot R = v \qquad \therefore\ v = -R\omega.$



iii) From the energy conservation,

$$mgh = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2$$

$$= \frac{1}{2}mv_0^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\cdot\left(-\frac{v_0}{R}\right)^2$$

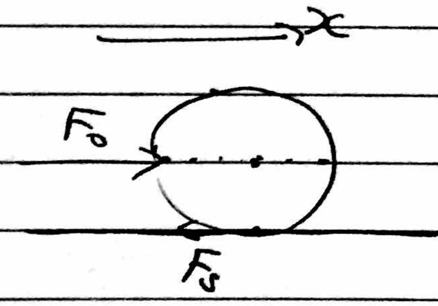
$$= \frac{3}{4}mv_0^2$$

$$\therefore\ h = \frac{3v_0^2}{4g} \approx 4.90\ [\mathrm{m}].$$

3.

iv) $I\alpha = -F_s \cdot R$,

$ma = F_0 - F_s$.



Also, $a = -R\alpha$

$$-F_s R = I\alpha = \frac{1}{2} m R^2 \cdot \left(-\frac{a}{R}\right) = \frac{-1}{2} m R a$$

$\rightsquigarrow \quad ma = 2F_s$.

$\longrightarrow \quad 2F_s = F_0 - F_s \qquad \therefore F_s = \frac{1}{3} F_0$.

Thus,

$$\alpha = -\frac{F_0 R}{3I} = -\frac{2F_0}{3mR}.$$

Now, $v : 0 \to v_0$ corresponds to $\omega : 0 \to -v_0/R$, and $\Delta\theta = -\frac{\Delta x}{R}$.

$$\omega_f^2 - \omega_i^2 = 2\alpha \Delta\theta$$

$$\Leftrightarrow \left(-\frac{v_0}{R}\right)^2 - 0^2 = 2\left(\frac{2F_0}{3mR}\right)\left(-\frac{\Delta x}{R}\right)$$

$$\therefore \Delta x = \frac{3mv_0^2}{4F_0} = 48 \text{ [m]}.$$