1. PHY9A Discussion 8 May 20, 2018

1.

i) $E_i = mgh_0 \approx 0.860$ [J].

$mgh_0 = \frac{1}{2}mv_0^2 \quad \therefore v_0 = \sqrt{2gh_0} \approx 5.42$ [m/s].

ii) $e = \frac{v_1}{v_0} \quad \therefore v_1 = e \cdot v_0 = e\sqrt{2gh_0} \approx 3.52$ [m/s].

$\Delta E_1 = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = (e^2-1)mgh_0 \approx -0.497$ [J].

iii) $e = \frac{v_n}{v_{n-1}} \rightsquigarrow e^n = \frac{v_n}{v_0} \quad \therefore v_n = e^n \cdot v_0 = e^n \cdot \sqrt{2gh_0}$.

$mgh_n = \frac{1}{2}mv_n^2 \quad \therefore h_n = \frac{v_n^2}{2g} = e^{2n} \cdot h_0$.

$\therefore h_3 = e^6 \cdot h_0 \approx 11.3$ [cm].

iv) $\bar{h} := 1$ [mm].

$\bar{h} > h_n = e^{2n} \cdot h_0 = \left(\frac{13}{20}\right)^{2n} \cdot h_0$

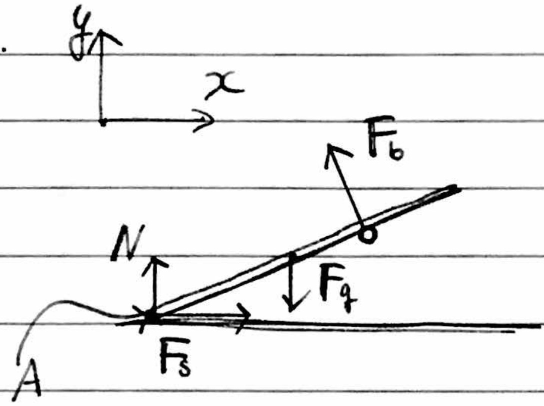
$\Leftrightarrow \left(\frac{20}{13}\right)^{2n} > \frac{h_0}{\bar{h}}$

$\therefore n > \frac{1}{2} \cdot \frac{\ln(h_0/\bar{h})}{\ln(20/13)} \approx 8.49$

$\therefore n \geq 9$.

2.

2\.
i)



The rod stays at rest, and thus the x-component of F_b must be canceled out by the friction in the positive x direction.

ii) Consider the total torque around the point A, and it must be zero:

$$\sum \tau = F_b \cdot l_b - F_g \cdot l_g \sin 60^\circ = 0.$$

where $l_b = h/\sin 30^\circ = 1.4$ [m], $l_g = \frac{l}{2} = 1$ [m], and $F_g = mg$.

$$\therefore F_b = F_g \cdot \frac{l_g}{l_b} \cdot \sin 60^\circ \approx 1.82 \text{ [N]}.$$

iii) The y-component of E.O.M:

$$0 = N - F_g + F_b \sin 60^\circ$$

$$= N - mg + mg \cdot \frac{l_g}{l_b} \cdot \frac{3}{4}$$

$$\therefore N = mg\left(1 - \frac{l_g}{l_b} \cdot \frac{3}{4}\right) = \frac{13}{28} \cdot mg.$$

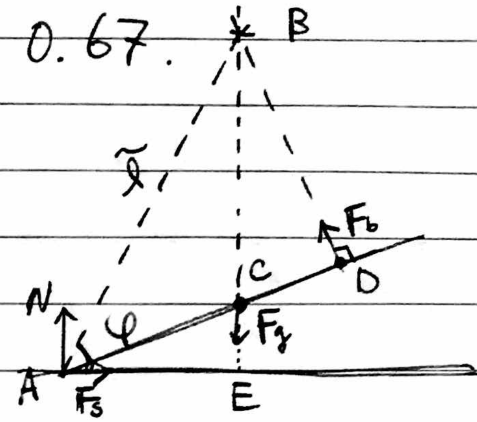
The x-component of E.O.M:

$$0 = F_s - F_b \cos 60^\circ$$

$$\therefore F_s = mg \cdot \frac{l_g}{l_b} \cdot \frac{\sqrt{3}}{4} = \frac{5\sqrt{3}}{28} \cdot mg \leq \mu_s N$$

3.

$$\therefore \ \mu_s \geqslant \frac{F_s}{N} = \frac{5\sqrt{3}}{13} \approx 0.67.$$



※ another way to solve this

Consider the total torque around the point B, and it also must be zero :

$$\begin{aligned}\Sigma \tau &= F_s \cdot \tilde{l} \cdot \sin\varphi - N \cdot \tilde{l} \cdot \sin(90 - \varphi) \\ &= F_s \cdot \tilde{l} \cdot \sin\varphi - N \cdot \tilde{l} \cdot \cos\varphi = 0.\end{aligned}$$

$$\therefore F_s = N \cdot \cot\varphi \leqslant \mu_s N$$

$$\therefore \ \mu_s \geqslant \cot\varphi$$

Then, we can geometrically find $\cot\varphi$:

$$\cot\varphi = \frac{l_{AE}}{l_{BE}} = \frac{l_g \cos 30^\circ}{l_{BC} + l_{CE}} = \frac{l_g \cos 30^\circ}{(l_b - l_g)\sec 60^\circ + l_g \sin 30^\circ}$$

$$= \frac{5\sqrt{3}}{13} \approx 0.67.$$

$$\therefore \ \mu_s \geqslant \frac{5\sqrt{3}}{13}.$$

iv) Yes, it could. If we change $l_g \to \frac{3}{2} = 1.5$ [m], we get

$$N = \frac{11}{56} \cdot mg,$$

and it's still positive. We also get

4.

$$F_s = \frac{15\sqrt{3}}{56} \cdot mg,$$

and

$$\mu_s \geqslant \frac{15\sqrt{3}}{11} \approx 2.4.$$

The situation requires high μ_s, but not impossible.