PHY 9A Discussion 6, Spring 2018

1. Circular Motion with Spring

One end of an ideal massless spring is fixed at the origin of the x-y coordinates on a horizontal plane, and a ball of mass $m = 2$ [kg] is attached to the other end of the spring. The spring has its spring constant $k = 5$ [N/m], and its original length $l_0 = 15$ [cm]. Now the ball is in a circular motion with a constant speed $v_0 = 20$ [m/s].

i. What is the length of the spring, that is, what is the radius of the circular motion?

ii. What is the total energy of the system?

iii. If you change the speed of the ball by a factor of two, how much does the length of the spring change? How about the total energy? (Assume that the ball is still in a circular motion after you have changed the speed.)

2. Potential Energy and Conservative Force

Calculate an associated potential energy if a force is given; calculate a corresponding conservative force if a potential energy is given. In the following, C is a constant with an appropriate unit, $\boldsymbol{e}_r$ is a unit vector into the radial direction, and $r = \sqrt{x^2 + y^2 + z^2}$. Assume that we are working in a 3-dimensional space. Remember that work done by a conservative force is guaranteed to be path-independent, and thus you can choose whatever convenient path for integration to calculate a potential energy.

(a) Conservative force:

$$\boldsymbol{F} = -C \cdot \boldsymbol{e}_y \,.$$

(b) Conservative force:

$$\boldsymbol{F} = \frac{C}{r^2}\boldsymbol{e}_r \,.$$

(c) Potential energy:

$$U = \frac{C}{r^3} \,.$$

(d) Potential energy:

$$U = C \cdot \frac{e^{-r/r_0}}{r} \,,$$

where $r_0 = 1$ [fm] ($= 10^{-15}$ [m]).

3. Analysis of Potential Energy

On the right is a graph of a mysterious potential energy of a massive object in one dimension.

i. Find equilibrium points.

ii. Where are the initial positions of the object that result in an unbounded motion? Assume that the particle is initially at rest, and use the work-energy principle to argue.

ex. Suppose that the x-axis in the graph shows a relative distance between the object and a potential source in a system, this potential actually does exist in our nature. Guess the system that has this potential energy. (Hint: When an object is orbiting around a potential source, we usually hit an infinite "potential wall" due to centrifugal potential as the relative distance gets smaller, but here we do not...)