1 PHY 9A Discussion 5 Apr. 28, 2018

1.

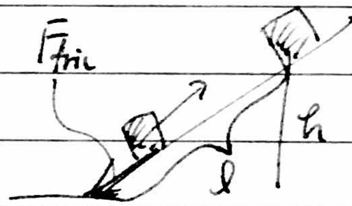
i) $E = KE + PE$.

$$E_i = KE_i + PE_i = \frac{mv_0^2}{2} + 0 = \frac{mv_0^2}{2}.$$

$$E_f = KE_f + PE_f = 0 + mgh = mgh.$$

ii) $F_{fric} = -\mu_k N = -\mu_k mg\cos\theta_{max}$.

$$l = \frac{h}{\sin\theta_{max}}$$



$$W_{ext} = F_{fric} \cdot l = -\mu_k mgh \frac{\cos\theta_{max}}{\sin\theta_{max}}$$

$$= -mgh \cdot \frac{\mu_k}{\tan\theta_{max}} = -mgh \cdot \frac{\mu_k}{\mu_s}.$$

iii) $\Delta E = E_f - E_i = mgh - \frac{mv_0^2}{2}$.

$$\therefore mgh - \frac{mv_0^2}{2} = -mgh \cdot \frac{\mu_k}{\mu_s}$$

$$\Leftrightarrow \left(1 + \frac{\mu_k}{\mu_s}\right) mgh = \frac{mv_0^2}{2}$$

$$\therefore h = \frac{v_0^2}{2g} \cdot \frac{1}{1 + \frac{\mu_k}{\mu_s}} = \frac{v_0^2}{2g} \cdot \frac{\mu_s}{\mu_s + \mu_k}.$$

2.

2.

i) $a_x = \frac{F_x}{m} = \frac{F_0}{m}(2\sin(2\omega t) - \sin(\omega t))$.

$\rightsquigarrow v_x = \int dt\, a_x = \frac{F_0}{m}\int dt\,(2\sin(2\omega t) - \sin(\omega t))$

$= \frac{F_0}{m\omega}(-\cos(2\omega t) + \cos(\omega t)) + v_{0,x}$

$= \frac{F_0}{m\omega}(\cos(\omega t) - \cos(2\omega t))$. $\left(\because 0 = v_x(t=0) = v_{0,x} \right)$

$\rightsquigarrow x = \int dt\, v_x = \frac{F_0}{m\omega}\int dt\,(\cos(\omega t) - \cos(2\omega t))$

$= \frac{F_0}{m\omega^2}\left(\sin(\omega t) - \frac{1}{2}\sin(2\omega t)\right) + x_0$

$= \frac{F_0}{m\omega^2}(1 - \cos(\omega t))\sin(\omega t)$. $\left(\because 0 = x(t=0) = x_0 \right)$

$a_y = \frac{F_y}{m} = \frac{F_0}{m}(2\cos(2\omega t) - \cos(\omega t))$

$\rightsquigarrow v_y = \int dt\, a_y = \frac{F_0}{m}\int dt\,(2\cos(2\omega t) - \cos(\omega t))$

$= \frac{F_0}{m\omega}(\sin(2\omega t) - \sin(\omega t)) + v_{0,y}$

$= \frac{F_0}{m\omega}(\sin(2\omega t) - \sin(\omega t))$. $\left(\because 0 = v_y(t=0) = v_{0,y} \right)$

$\rightsquigarrow y = \int dt\, v_y = \frac{F_0}{m\omega}\int dt\,(\sin(2\omega t) - \sin(\omega t))$

3.

$$= \frac{F_0}{m\omega^2}\left(-\frac{1}{2}\cos(2\omega t) + \cos(\omega t)\right) + y_0$$

$$= \frac{F_0}{m\omega^2}\left(\cos(\omega t) - \frac{1}{2}\left(2\cos^2(\omega t) - 1\right)\right) + y_0$$

$$= \frac{F_0}{m\omega^2}\left(1 - \cos(\omega t)\right)\cos(\omega t) + \left(\frac{F_0}{2m\omega^2} + y_0\right)$$

$$= \frac{F_0}{m\omega^2}\left(1 - \cos(\omega t)\right)\cos(\omega t). \quad \left(\because\ 0 = y(t=0) = \frac{F_0}{2m\omega^2} + y_0\right)$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{F_0}{m\omega^2}\begin{pmatrix} (1-\cos(\omega t))\cdot\sin(\omega t) \\ (1-\cos(\omega t))\cdot\cos(\omega t) \end{pmatrix}.$$

ii) Yes.

$$x = 0 \Rightarrow \begin{array}{l} \cos(\omega t) = 1 \\ \text{or } \sin(\omega t) = 0 \end{array} \Rightarrow \begin{array}{l} \omega t = 2\pi n \\ \text{or } \omega t = \pi n \end{array}$$

$$\Rightarrow t = n \cdot \frac{\pi}{\omega} \quad (n \in \mathbb{Z}_{\geq 0}).$$

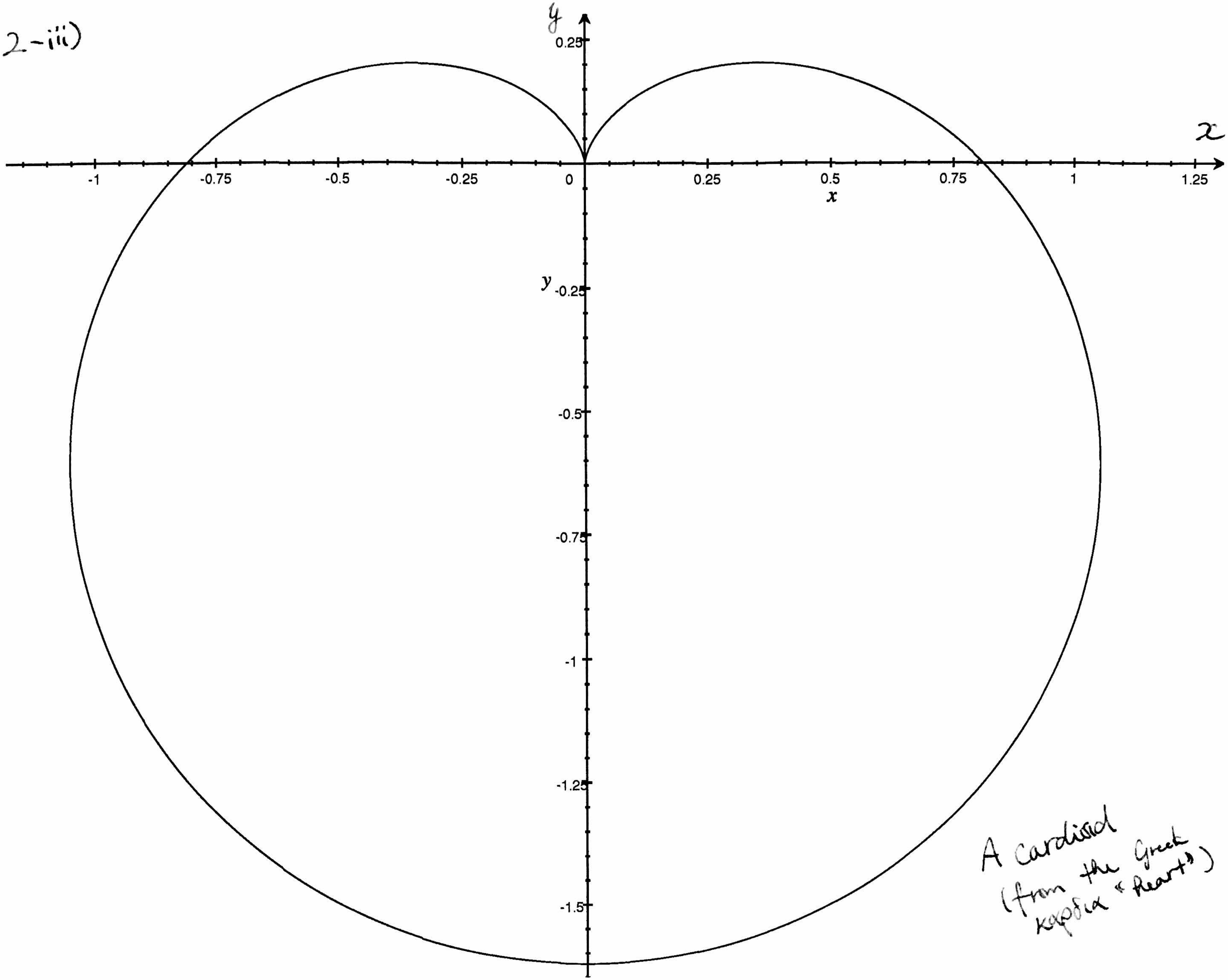
$$y = 0 \Rightarrow \begin{array}{l} \cos(\omega t) = 1 \\ \text{or } \cos(\omega t) = 0 \end{array} \Rightarrow \begin{array}{l} \omega t = 2\pi n \\ \text{or } \omega t = (n+\frac{1}{2})\pi \end{array}$$

$$\Rightarrow t = 2n \cdot \frac{\pi}{\omega}, \text{ or } \left(n+\frac{1}{2}\right)\cdot\frac{\pi}{\omega} \quad (n \in \mathbb{Z}_{\geq 0})$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow t_n = 2n \cdot \frac{\pi}{\omega} \quad (n \in \mathbb{Z}_{\geq 0}).$$

$$t_1 = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/2} = 4 \ [s].$$

2-iii)



A cardioid
(from the Greek
καρδια « heart »)