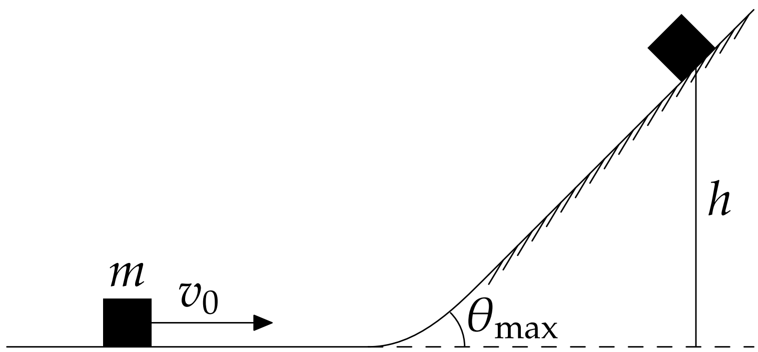
PHY 9A Discussion 5, Spring 2018

1. Box on a Slope with Friction — Revisited

A box of mass $m = 10$ [kg] is moving on a frictionless ground in the horizontal direction with constant speed $v_0 = 8$ [m/s], and starts to climb up a slope, which is smoothly connected to the frictionless ground, with an angle, θ, to the horizontal at $t = 0$ [s]. Unlike the ground, the slope has the static frictional coefficient of $\mu_s = 0.4$ and the kinetic frictional coefficient of $\mu_k = 0.1$.

In the previous discussion, we have seen the maximum angle with which the box stops and stays at the highest position is given by $\tan(\theta_{max}) = \mu_s$. By using $\boldsymbol{F} = m\boldsymbol{a}$, we have also calculated the height, h, to the position of the box when it stops with $\theta = \theta_{max}$, and found

$$h = \frac{{v_0}^2}{2g} \frac{\mu_s}{\mu_s + \mu_k} .$$



Here we calculate h again, but with the work-energy principle, and we will find it is much easier. In the following questions, consider the case where $\theta = \theta_{max}$.

i. Find the initial and final mechanical energies.

ii. Calculate the energy dissipated due to kinetic friction.

iii. By using the (generalized) work-energy principle, $\Delta E = W_{ext}$, calculate h.

2. Time-dependent Force

A time-dependent force in 2-dimensional flat space expressed in the cartesian coordinates,

$$\boldsymbol{F}(t) = \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} = F_0 \begin{pmatrix} 2\sin(2\omega t) - \sin(\omega t) \\ 2\cos(2\omega t) - \cos(\omega t) \end{pmatrix} ,$$

where $F_0 = 10$ [N], and $\omega = \pi/2$ [rad/s], is acting on a test particle with mass $m = 5$ [kg], and the particle is at rest at $t = 0$ [s].

i. Find the position of the particle as a function of time.

ii. Does the particle come back to its original position? If yes, then find the time it first comes back; if no, then find the position of the particle at $t = 4$ [s].

iii. Try to draw the trajectory of the particle. (You can use your favorite graphing software to do so.) What does it look like?