1. PHY9A Discussion 4 Apr. 23. 2018

1.

i) $\left.\frac{v_y}{v_x}\right|_{t=T} = \tan 45° = 1$, $v_x = v_0$ ($\because a_x = 0$).

$\therefore \quad v_y(t=T) = v_0$.

Here, we set $t=0$ as the time the ball enters the region, and $t=T$ the time the ball leaves the region.

$x = v_0 t + x_0 \rightsquigarrow \Delta x = v_0 t \qquad \therefore T = \frac{\Delta x}{v_0}$.

$v_y = a_y t \rightsquigarrow v_0 = a_y T \qquad \therefore a_y = \frac{v_0}{T} = \frac{v_0^2}{\Delta x}$.

Now, $ma_y = F - mg$

$\therefore F = m(a_y + g) = m\left(\frac{v_0^2}{\Delta x} + g\right) \simeq 102.9$ [N].

ii) $\left.\frac{v_y}{v_x}\right|_{t=T} = \tan 45° = 1$, $\sqrt{v_x^2 + v_y^2} = v_0$.

$\therefore \quad v_x(t=T) = v_y(t=T) = \frac{v_0}{\sqrt{2}}$.

$$\begin{cases} x = \frac{a_x}{2}t^2 + v_0 t + x_0 \\ v_x = a_x t + v_0 \end{cases}$$

$\rightsquigarrow \quad \Delta x = \frac{a_x t}{2} \cdot t + v_0 t = \frac{v_x - v_0}{2} t + v_0 t$

$= \frac{v_x + v_0}{2} \cdot t$.

2.

$$\therefore \Delta x = \frac{v_x(t=T) + v_0}{2} \cdot T = \frac{v_0/\sqrt{2} + v_0}{2} \cdot T$$

$$= \frac{1+\sqrt{2}}{2\sqrt{2}} \cdot v_0 T$$

$$\Longleftrightarrow T = \frac{\Delta x}{v_0} \cdot \frac{2\sqrt{2}}{1+\sqrt{2}} .$$

$$a_x = \frac{v_x(t=T) - v_0}{T} = \frac{v_0}{\Delta x} \cdot \frac{1+\sqrt{2}}{2\sqrt{2}} \cdot \left(\frac{v_0}{\sqrt{2}} - v_0 \right)$$

$$= \frac{v_0^2}{\Delta x} \cdot \frac{1+\sqrt{2}}{2\sqrt{2}} \cdot \frac{1-\sqrt{2}}{\sqrt{2}} = -\frac{v_0^2}{4\Delta x} .$$

$$\therefore F_x = m a_x = -\frac{m v_0^2}{4\Delta x} \quad (\simeq -18.38 \text{ [N]}).$$

$$a_y = \frac{v_y(t=T)}{T} = \frac{v_0}{\Delta x} \cdot \frac{1+\sqrt{2}}{2\sqrt{2}} \cdot \frac{v_0}{\sqrt{2}}$$

$$= \frac{v_0^2}{4\Delta x} (1+\sqrt{2}) .$$

$$\therefore F_y = m(a_y + g) = m\left(\frac{v_0^2}{4\Delta x}(1+\sqrt{2}) + g \right)$$

$$(\simeq 73.76 \text{ [N]}).$$

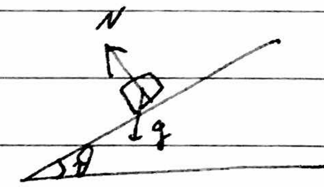
$$\rightsquigarrow F = \sqrt{F_x^2 + F_y^2} \simeq 76.02 \text{ [N]}.$$

$$\theta_F = \arctan\left(\frac{F_y}{F_x} \right) = \arctan\left(-\left(1+\sqrt{2} + \frac{4\cdot\Delta x \cdot g}{v_0^2}\right)\right)$$

$$\simeq 104.0^\circ$$

3.

2.



i) $F_f \leqq mg\cos\theta \cdot \mu_s$,

$F_g = -mg\sin\theta$

$$0 = F_f + F_g \leqq mg\cos\theta \cdot \mu_s - mg\sin\theta$$

$$\therefore \tan\theta \leqq \mu_s$$

$$\therefore \theta_{max} = \arctan(\mu_s) \approx 21.80^\circ$$

Along the slope,

$$l = \frac{a}{2}t^2 + v_0 t, \quad v = at + v_0,$$

$$ma = -mg\cos\theta \cdot \mu_k - mg\sin\theta \quad \therefore a = -g(\mu_k\cos\theta + \sin\theta).$$

$$0 = at^* + v_0 \quad \therefore t^* = -\frac{v_0}{a} = \frac{v_0}{g(\mu_k\cos\theta + \sin\theta)}$$

$$\therefore l = \frac{a}{2}t^{*2} + v_0 t^* = \frac{a}{2}\left(\frac{v_0}{a}\right)^2 - v_0\frac{v_0}{a} = -\frac{v_0^2}{2a}$$

$$= \frac{v_0^2}{2g(\mu_k\cos\theta + \sin\theta)}$$

$$\therefore h = l\sin\theta_{max} = \frac{v_0^2\sin\theta_{max}}{2g(\mu_k\cos\theta_{max} + \sin\theta_{max})}$$

$$= \frac{v_0^2\tan\theta_{max}}{2g(\mu_k + \tan\theta_{max})} = \frac{v_0^2}{2g}\cdot\frac{\mu_s}{\mu_k + \mu_s}$$

$$\approx 2.612 \ [m]$$

4.

3.

i) $\omega = 2\pi f$, $\quad v = r\omega = 2\pi r f \approx 18.85$ [m/s].

ii) Schematically, we have

$$m \cdot \frac{v^2}{r} \sim (\text{string tension}) + (\text{gravity}).$$

- The gravity term is constant
  $\rightsquigarrow$ As v gets larger, the tension gets larger, and the gravity term becomes relatively negligible.
- The tension term cannot be negative
  $\rightsquigarrow$ As v gets smaller, the tension gets smaller, and at some $v = \tilde{v}$, the tension becomes zero. With $v < \tilde{v}$, the motion fails to be circular, and, with even smaller v, it starts to be oscillatory.