1. PHY 9A Discussion 2 Apr. 8, 2018

1.

i) $x = v_0 \cos\theta \cdot t$

$y = v_0 \sin\theta \cdot t - \frac{g}{2} t^2$.

$y = 0 \Leftrightarrow t = 0, \frac{2v_0 \sin\theta}{g}$.

$\therefore x = \frac{2v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2}{g} \sin(2\theta)$.

$\theta = 45°$ gives the maximum x of $\frac{v_0^2}{g} \approx 10.2$ [m]

ii) $v_y = v_0 \sin\theta - gt = 0$, $\therefore t = \frac{v_0 \sin\theta}{g}$

$\rightsquigarrow y = v_0 \sin\theta \cdot \frac{v_0 \sin\theta}{g} - \frac{g}{2} \cdot \left(\frac{v_0 \sin\theta}{g}\right)^2 = \frac{v_0^2 \sin^2\theta}{2g}$.

When $\theta = 45°$, $y = \frac{v_0^2}{4g} \approx 2.55$ [m].

iii) Since $\sin(90° + \theta) = \cos(\theta)$,

$x_\pm = \frac{v_0^2}{g} \sin(2(45° \pm \Delta\theta)) = \frac{v_0^2}{g} \sin(90° \pm 2\Delta\theta)$

$= \frac{v_0^2}{g} \cos(\pm 2\Delta\theta) = \frac{v_0^2}{g} \cos(2\Delta\theta)$.

$\therefore x_+ = x_- \left(= \frac{v_0^2}{g} \cos(2\Delta\theta)\right)$.

But, $t_\pm = \frac{2v_0}{g} \sin(45° \pm \Delta\theta)$, and

$t_+ > t_-$.

2.

2.

i) $v_B = \begin{cases} 10 & (0 \le t \le 4), \\ 6 & (4 < t). \end{cases} \Rightarrow x_B = \begin{cases} 10t & (0 \le t \le 4), \\ 40 + 6(t-4) & (4 < t). \end{cases}$

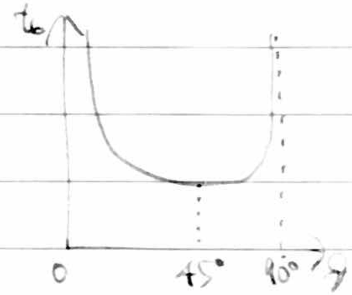
$x_B = 100 \Rightarrow t_B = 14$ [s].

ii) The acceleration along the ramp is $g\sin\theta$.

$\rightsquigarrow$ The x-component : $a_x = g\sin\theta \cdot \cos\theta$.

$$x_b = \frac{g}{2}\sin\theta \cdot \cos\theta \cdot t^2 = \frac{g\sin(2\theta)}{4} t^2 .$$

$$x_b = 100 \Rightarrow t_b = \sqrt{\frac{400}{g\sin(2\theta)}} .$$



iii) $t_b = \sqrt{\frac{400}{g\sin(2\theta)}} \Longleftrightarrow \theta = \frac{1}{2}\arcsin\left(\frac{400}{g t_b^2}\right)$.

$t_b = 14 \rightarrow \theta \cong 6.01°$.

t_b is bounded from below, and minimized by $\theta = 45°$, thereby becoming $t_b \cong 6.39$ [s]. Therefore, if Bob can run so that his record time becomes smaller than 6.39 [s], then he wins the race regardless of θ. (It is not realistic, though...)