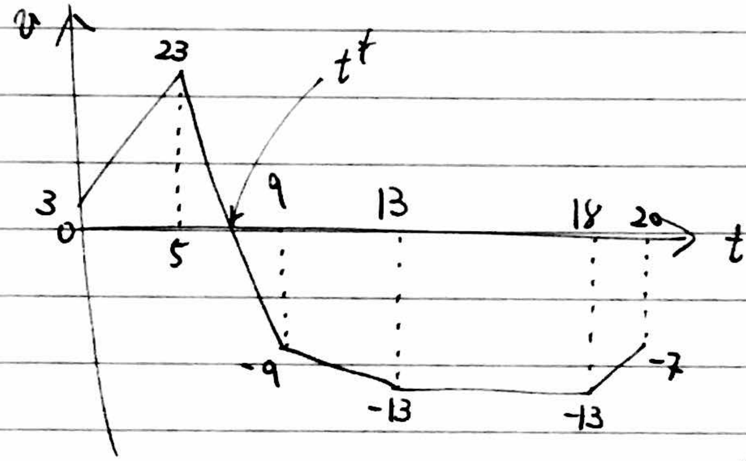
1. PHY 9A Discussion 1 Apr. 5, 2018

1.

i)



$v = v_0 + a(t - t_0)$

v can be zero only in [5, 9].

$\rightsquigarrow v_2 = v_1(t=5) + a_2(t-5)$

$= 23 - 8(t-5) = 0$ $\therefore t = t^* = 7.875$ [s].

ii)

t	0–5	5–t^*	t^*–9	9–13	13–18	18–20
v	+	+	−	−	−	−
a	+	−	−	−	0	+

Different directions.

iii) Using $x = x_0 + v_0(t-t_0) + \frac{a}{2}(t-t_0)^2$,

$x_1 = 3t + 2t^2,$ $(0 \le t \le 5)$

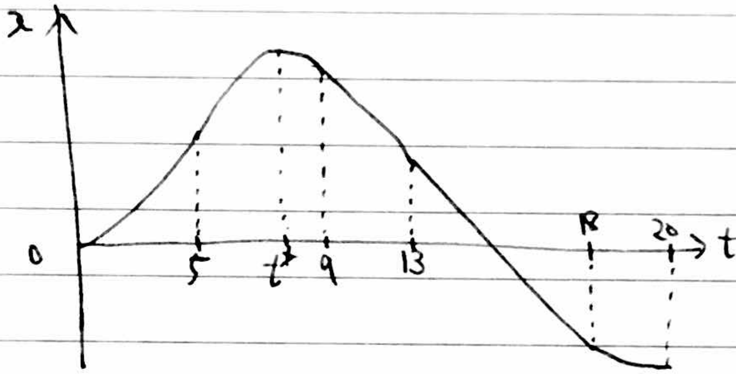
$x_2 = 65 + 23(t-5) - 4(t-5)^2$ $(5 \le t \le 9)$

$x_3 = 93 - 9(t-9) - \frac{1}{2}(t-9)^2$ $(9 \le t \le 13)$

$x_4 = 49 - 13(t-13)$ $(13 \le t \le 18)$

$x_5 = -16 - 13(t-18) + \frac{3}{2}(t-18)^2$ $(18 \le t \le 20)$

2.

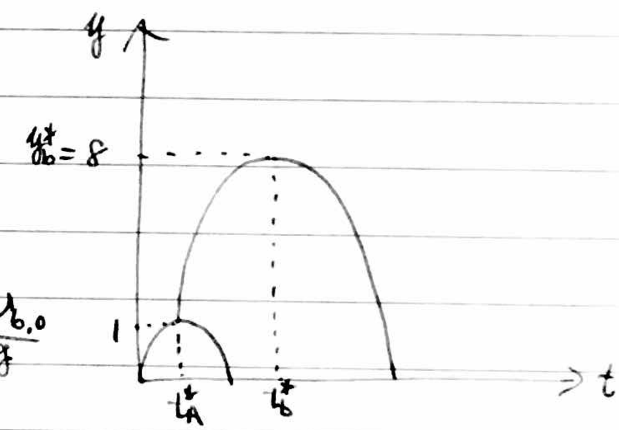


∘Farthest positive x : $x_2(t=t^*) = 98.0625$ [m]

∘Farthest negative x : $x_5(t=20) = -36$ [m]

∴ The farthest point is 98.0625 [m] away.

2.



i) $v_b = v_{b,0} - g(t - t_A^*)$,

$y_b = y_{b,0} + v_{b,0}(t - t_A^*) - \frac{g}{2}(t - t_A^*)^2$.

$0 = v_{b,0} - g(t_b^* - t_A^*)$, ∴ $t_b^* - t_A^* = \frac{v_{b,0}}{g}$

$\rightsquigarrow 8 = 1 + v_{b,0} \cdot \frac{v_{b,0}}{g} - \frac{g}{2}\left(\frac{v_{b,0}}{g}\right)^2$

$= 1 + \frac{v_{b,0}^2}{2g}$

∴ $v_{b,0} = \sqrt{2g(y_b^* - y_{b,0})}$

$= \sqrt{2 \cdot 9.8 \cdot (8-1)} \approx 11.71$ [m/s].

ii) $v_A = v_{A,0} - gt$,

$y_A = v_{A,0}t - \frac{g}{2}t^2$.

Similarly, $v_{A,0} = \sqrt{2g y_A^*} = \sqrt{2g}$.

$0 = v_{A,0}t - \frac{g}{2}t^2 = t(v_{A,0} - \frac{g}{2}t)$ ∴ $t = 0, \frac{2v_{A,0}}{g}$

$\rightsquigarrow$ At $t = \frac{2v_{A,0}}{g}$, Alice touched the ground.

Noticing $t_A^* = \frac{v_{A,0}}{g}$, $t - t_A^* = \frac{v_{A,0}}{g}$, and

3.

$$y_b = y_{b,0} + v_{b,0} \cdot \frac{v_{A,0}}{g} - \frac{g}{2} \cdot \left(\frac{v_{A,0}}{g}\right)^2$$

$$= 1 + \sqrt{2g(8-1)} \cdot \sqrt{\frac{2}{g}} - \frac{g}{2}\left(\sqrt{\frac{2}{g}}\right)^2 = 2\sqrt{7} \approx 5.292 \ [m],$$

$$v_b = v_{b,0} - g \cdot \frac{v_{A,0}}{g} = \sqrt{2g}\,(\sqrt{7} - 1) \approx 7.286 \ [m/s] > 0$$

$\rightsquigarrow$ Going up.

iii) We had $y_b^* = y_{b,0} + \frac{v_{b,0}^2}{2g}$. Let $T\ (< t_A^*)$ be the time at which Alice throws a ball. Then, we have to change

$$y_{b,0} \longmapsto y_A(t=T), \quad v_{b,0} \longmapsto v_{b,0} + v_A(t=T),$$

and we get

$$y_b^* = y_A(t=T) + \frac{(v_{b,0} + v_A(t=T))^2}{2g}$$

$$= v_{A,0}T - \frac{g}{2}T^2 + \frac{v_{b,0}^2 + v_A(t=T)^2 + 2v_{b,0} \cdot v_A(t=T)}{2g}$$

$$= v_{A,0}T - \frac{g}{2}T^2 + \frac{v_{b,0}^2}{2g} + \frac{(v_{A,0} - gT)^2}{2g} + \frac{v_{b,0}}{g}(v_{A,0} - gT)$$

$$= \cancel{v_{A,0}T}^{①} - \cancel{\frac{g}{2}T^2}^{②} + \frac{v_{b,0}^2}{2g} + \underbrace{\frac{v_{A,0}^2}{2g}}_{= y_{b,0}} + \cancel{\frac{g}{2}T^2}^{②} - \cancel{v_{A,0}T}^{①} + \frac{v_{b,0}v_{A,0}}{g} - v_{b,0}T$$

$$= \underbrace{y_{b,0} + \frac{v_{b,0}^2}{2g}}_{= 8} + v_{b,0}\Big(\underbrace{\frac{v_{A,0}}{g}}_{= t_A^*} - T\Big)$$

$$= 8 + v_{b,0}(t_A^* - T)$$

$$> 8 \quad (\because T < t_A^*).$$

$\therefore$ The ball would have gone higher than 8 [m].