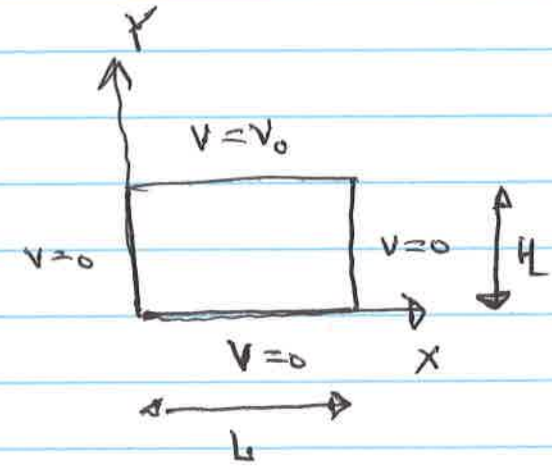
L1

Laplace Eqn



$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$V(x,y) = f(x)g(y)$$

$$g\frac{d^2 f}{dx^2} + f\frac{d^2 g}{dy^2} = 0$$

$$\frac{1}{g}\frac{d^2 g}{dy^2} = -\frac{1}{f}\frac{d^2 f}{dx^2} = k^2$$

$$g(y) = \begin{cases} e^{ky} \\ e^{-ky} \end{cases} \qquad f(x) = \begin{cases} \sin kx \\ \cos kx \end{cases}$$

equivalently $\begin{cases} \cosh ky \\ \sinh ky \end{cases}$

We want $V=0$ for $x=0$ so choose $\sin kx$

$V=0$ for $y=0$ so choose $\sinh ky$

furthermore $V=0$ for $x=L$ $\Rightarrow$ $k = \frac{n\pi}{L}$

$$V(x,y) = \sum_n a_n \sin\frac{n\pi x}{L} \sinh\frac{n\pi y}{L}$$

L2

The final bdy condition is $V(x, y=H) = V_0$

$$V_0 = \sum_n a_n \sin\frac{n\pi x}{L} \sinh\frac{n\pi H}{L}$$

$$\int_0^L \sin\frac{m\pi x}{L} V_0 \, dx = \sum_n a_n \sinh\frac{n\pi H}{L} \underbrace{\int_0^L \sin\frac{m\pi x}{L} \sin\frac{n\pi x}{L} dx}_{\frac{L}{2}\delta_{nm}}$$

$$= \frac{L}{2} a_m \sinh\frac{m\pi H}{L}$$

$$V(x,y) = \sum_n a_n \sin\frac{n\pi x}{L} \sinh\frac{n\pi y}{L}$$

$$a_n = \frac{2}{L} \frac{V_0}{\sinh\frac{n\pi H}{L}} \underbrace{\int_0^L \sin\frac{n\pi x}{L} dx}$$

$$-\frac{L}{n\pi} \cos\frac{n\pi x}{L} \Big|_0^L$$

$$-\frac{L}{n\pi} (\underbrace{\cos n\pi}_{(-1)^n} - 1)$$

n even: $\rightarrow 0$

n odd: $2L/n\pi$

L3

Final Answer:

$$\boxed{V(x,y) = \sum_{n=1,3,5,\ldots} \frac{4V_0}{n\pi \sinh \frac{n\pi H}{L}} \sin \frac{n\pi x}{L} \sinh \frac{n\pi y}{L}}$$

write C code to compute $V(x,y)$ for

given input x, y, V_0