Physics 40: Laboratory Six

Thursday, April 16, 2020

Today's Goals: Molecular dynamics in $d = 2$: projectile motion and the Kepler Problem.

[0] Review of MD in 2D; Review of Kepler.

[1] Write a molecular dynamics code for 'projectile motion' (a mass moving under the constant downwards force of gravity near the surface of the earth). The basic ideas are the same as for the $d = 1$ problem. See discussion on blackboard, partially reproduced here:

Instead of a one dimensional position x, we now have x, y.

Instead of v, we now have vx, vy.

Instead of a, we now have ax, ay (determined by the components of the force in the x, y directions).

So the 'heart of your code' looks like this:

```
for (i=1; i<N+1; i=i+1)
{
     x=x+vx*dt;
     y=y+vy*dt;
     ax=0.;
     ay=-9.8;
     vx=vx+ax*dt;
     vy=vy+ay*dt;
     t=t+dt;
     fprintf(fileout,"\n%12.6lf %12.6lf",x,y);
 }
```

Notice we are choosing to plot (x, y) instead of (t, x).

Complete the above MD code. (Note: I am asking you to do more on your own here. You need to add in the 'include ⟨stdio.h⟩' and 'int main()', the declarations of variables, etc. etc.)

[PS3-3] Run your projectile motion code for initial position $x = 0, y = 0$; initial velocity $vx = 5, vy = 19.6$. mass $m = 5$. What is the 'range'? That is, how far does the mass travel before it hits the ground ($y = 0$)? What happens if $m = 7$? Does this answer agree with what you would get using your classical mechanics formulae?

[PS3-4] Your projectile motion code works for a mass near the surface of the earth. But gravity is a force which also determines the motion of the earth around the sun (and of stars in the Milky way about the black hole at the center of our galaxy $\cdots$). Write a MD code for the 'Kepler Problem' of the earth orbiting the sun. You might find the discussion on the blackboard, where we review "Newton's universal law of gravity" and determine the forces in the x, y directions, useful.

You might also need:

```
G      = 6.67 x 10^{-11}  m^{3}/(kg s^{2})
Msun   = 2.0 x 10^{30}  kg
Mearth = 6.0 x 10^{24}  kg
R      = 1.5 x 10^{11}  m     (distance from earth to sun)
V      = 3.0 x 10^{4}   m/s   (velocity of earth around sun)
```

Run your code starting the earth off at $x = R$ and $y = 0$, and with $vx = 0, vy = V$.
(a) Why is vy nonzero and not vx?
(b) What is a good choice of time step dt for this problem?
(c) Run your code with a number of steps N corresponding to 9 months. Make a plot of the resulting orbit. Is it correct?
(d) Run your code with the same parameters except $vx = 0, vy = V/2$. Make a plot of the resulting orbit. Comment.
(e) Run your code with the same parameters except $vx = 0, vy = 2V$. Make a plot of the resulting orbit. Comment. For this problem you might want to see what happens as you increase N.