```c
#include <stdio.h>
#include <math.h>
int main(void)
{
    FILE * fileout;
    int x,t,Nt;
    double rho[1000],newrho[1000],D,norm;

    fileout=fopen("slughorn.txt","w");

    printf("\nEnter D*dt/dx^2   ");
    scanf("%lf",&D);
    printf("\nEnter number of time steps   ");
    scanf("%d",&Nt);

    for (x=0; x<1000; x=x+1)
    {
        rho[x]=0.0;
    }
    rho[500]=1.0;

    for (t=0; t<Nt; t=t+1)
    {
        for (x=1; x<999; x=x+1)
        {
            newrho[x]=rho[x]+D*(rho[x+1]+rho[x-1]-2.0*rho[x]);
        }
        norm=0.0;
        for (x=1; x<999; x=x+1)
        {
            rho[x]=newrho[x];
            norm=norm+rho[x];
        }
        printf("\n checking no particles are lost! norm= %8.3lf  ",norm);
    }

    for (x=0; x<1000; x=x+1)
    {
        fprintf(fileout,"\n %6d %8.4lf",x,rho[x]);
    }

    fclose(fileout);
    printf("\n");
    return 0;
}
```

Bad notation $D = D\frac{dt}{(dx)^2}$

initialization, like x_0 y_0 vx_0 vy_0 in Kepler

loop within loop

why use newrho? HW question

resetting for next pass through loop

Speed estimation

\# operations: $Nt \cdot Nx \cdot 10$ roughly

$CPU = 3\,GHz = 3 \cdot 10^9$ ops/sec

$Nt = 10^5$ $Nx = 10^3$

CPU time $\sim \frac{1}{3}$ second

DE-1

Diffusion Equation Discussion

$$p[x] = p[x] + D\left(p[x+1] + p[x-1] - 2p[x]\right)$$

$D = 1/2 \qquad p[500] = 1$

$x =$	497	498	499	500	501	502	503
$p[x] =$	0	0	0	1	0	0	0
	0	0	1/2	0	1/2	0	0
	0	1/4	0	1/2	0	1/4	0
	1/8	0	3/8	0	3/8	0	1/8

Pascal triangle

1	1
1 1	1/2
1 2 1	1/4
1 3 3 1	1/8
1 4 6 4 1	1/16

DE-2

Random walk: Toss fair coin

H: step left   T: step right

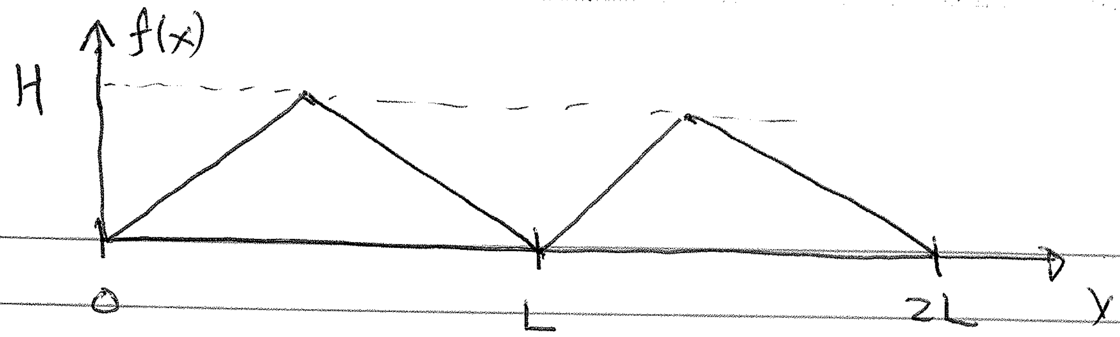
Start at 500

$X =$	499	501
prob	1/2	1/2
	H	T

HH	498	1/4
HT TH	500	1/2
TT	502	1/4

Diffusion $\equiv$ Random Walk

2-3



Fourier series $\quad f(x) = f(x+L)$ periodic

$$f(x) = a_0 + a_1 \cos\frac{2\pi x}{L} + a_2 \cos\frac{4\pi x}{L} + \dots$$
$$+ b_1 \sin\frac{2\pi x}{L} + b_2 \sin\frac{4\pi x}{L} + \dots$$

$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx = \text{average value of } f$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{2\pi n x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{2\pi n x}{L}\,dx$$

Fourier integral: Any $f(x)$

$$f(x) = \int_{-\infty}^{\infty} c(k)\, e^{ikx}\, dk$$

$$c(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx / 2\pi$$

DE 4

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2}$$

$$p(x,t) = f(x)\, g(t)$$ Separation of variables

$$f(x) \frac{dg}{dt} = D \frac{d^2 f}{dx^2} g(t)$$

$$\frac{1}{f} \frac{d^2 f}{dx^2} = \frac{1}{Dg} \frac{dg}{dt} = \text{const} = -k^2$$

$$g(t) = e^{-Dk^2 t}$$

$$f(x) = e^{ikx}$$

$$p(x,t) = e^{ikx} e^{-Dk^2 t}$$

$$p(x,t) = \int_{-\infty}^{\infty} c(k)\, e^{ikx} e^{-Dk^2 t}\, dk$$

$$p(x,0) = \int_{-\infty}^{\infty} c(k)\, e^{ikx}\, dk$$

$$c(k) = \int_{-\infty}^{\infty} p(x,0)\, e^{-ikx}\, dx / 2\pi$$

$$p(x,0) = \delta(x) \qquad c(k) = 1/2\pi$$

$$p(x,t) = \int \frac{1}{2\pi} e^{ikx} e^{-Dk^2 t}\, dk$$

DE-5

$$p(x,t) = \frac{1}{2\pi}\overbrace{\int dk\, e^{-Dt\left(k - \frac{ix}{2Dt}\right)^2}}^{\sqrt{\pi/Dt}} e^{-x^2/4Dt}$$

$$\boxed{p(x,t) = \sqrt{\frac{1}{4\pi Dt}}\, e^{-x^2/4Dt}}$$

analytic soln.

$D = .02$

$dt = .0001$ $\qquad D\, dt/dx^2 = 0.02$

$dx = .01$

$N = 100000$ $\qquad t = 10$ $\qquad 4Dt = 0.8$

$x = \sqrt{.8} \simeq .9$